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GEOMETRICAL MODELING OF THE PROCESS OF WEAVING A WIRE CLOTH IN WEIGHTLESSNESS USING THE INERTIAL UNFOLDING OF A DUAL PENDULUM

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Запропоновано спосіб виготовлення у невагомості металевого сіткополотна за допомогою коливань ряду подвійних маятників. Коливання виникають завдяки впливу на вузли елементів маятника імпульсів двох реактивних двигунів, тим самим забезпечуючи його інерційне розкриття. Опис процесу інерційного розкриття маятника виконано за допомогою рівняння Лагранжа другого роду. Результати доцільно використати при проектуванні масштабних сіткополотен, наприклад активних поверхонь антен довгохвильового діапазону, та їх виготовлення в умовах невагомості

Ключові слова: геометричне моделювання, сіткополотно, подвійний маятник, розкриття антени, рівняння Лагранжа другого роду

Предложен способ изготовления в невесомости металлического сетеполотна при помощи колебаний ряда двухзвенных маятников. Колебания возникают благодаря влиянию на узлы элементов маятника импульсов двух реактивных двигателей, тем самым обеспечивая его инерционное раскрытие. Описание процесса инерционного раскрытия маятника выполнено с помощью уравнения Лагранжа второго рода. Результаты целесообразно использовать при проектировании масштабных сетеполотен, например активных поверхностей антенн длинноволнового диапазона, и их изготовления в условиях невесомости

Ключевые слова: геометрическое моделирование, сетеполотно, двойной маятник, раскрытие антенны, уравнение Лагранжа второго рода

1. Introduction

Modern trends in the development of space systems for telecommunication necessitate creation of large and highly efficient antennas [1]. In most cases, reflector antennas are shaped in the form of a rigid frame that can be transformed and thus take on the calculated design form. The frame holds a flexible radio-reflective surface, which is made of metal woven wire cloth, that is, a material woven from thin metal wires [2, 3]. It is known that a parabolic antenna with a diameter of about 3 meters was deployed on the surface of the Moon during flight of the American spacecraft Apollo 11. Its surface was fabricated from the woven mesh, made of steel wire covered with gold.

The starting material used at weaving a metal wire cloth includes gold plated steel, tungsten, molybdenum, copper, and other threads with a diameter of 30...90 μm with the size of core cells ranging from 1 mm to 3 mm. Such materials are to be used in the advanced designs of space large-diameter antennas that can be transformed. Radio-reflective surface of a woven wire cloth must possess high coefficient of radio wave reflection in the working frequency range. In addition, a woven structure should meet requirements for minimal stretching effort, high stability of physical-mechanical and electrical-physical characteristics during storage, as well as long-term operation [4].

Space antennas operate in a wide range of wavelengths and frequencies. There is a relationship between length of the electromagnetic wave and size of the cells in a woven core of a metal woven wire cloth of the reflecting surface of antenna. To achieve high coefficient of electromagnetic wave reflection, the size of cells in a woven wire cloth should be 15–20 times smaller than the wavelength [5]. The structure of the woven wire cloth is chosen according to the preset parameters of electromagnetic wave. An increase in the frequency of electromagnetic wave leads to a decrease in wavelength, which requires the use of woven wire cloth with smaller and smaller core size.

On the other hand, a decrease in the frequency of electromagnetic wave increases the wavelength with the possibility to use woven wire cloth with a larger size of core cells and larger diameter of wire. But the problem arises on enabling the construction of large-scale space antennas with length of tens of meters or even kilometers. This makes it possible to use woven wire cloth with considerable core cell size. Large-sized structures are transported at present into orbit folded, to subsequently unfold in order to acquire the designed shape. Control over unfolding of large structures in space is a complex scientific and technical challenge of mechanics, which has no analogues in ground-based equipment [6]. In addition, creation of large-scale structures that are transformed in space is associated with resolving several problems in engineering and mechanics due to unique objects. Their characteristic feature is the combination of conflicting requirements regarding a significant increase in their dimensions while ensuring sufficient rigidity at a rather limited mass of the force frame.

The way out of such a circle of contradictions implies development of the technique for constructing large-scale woven wire cloth directly in the orbit in space. Thus, it is an important task to devise a method for weaving a wire cloth in weightlessness. The initial stage of tackling it should be a clear and correct geometrical model of shape-formation of spatial patterns in the elements of a woven wire cloth.

2. Literature review and problem statement

At present, all bulky components of spacecraft are designed so that they can be made on Earth, to be subsequently folded to fit a rocket and then unfold in the orbit. It is clear that such an approach imposes limitations to the production of components, primarily in terms of excessive strength margin, and requires larger financial investment. Existing contemporary techniques are directed mainly to the implementation of such a framework. Paper [7] lays out a general approach to the construction of mathematical models for the analysis of the process of unfolding the structures of space antennas. Article [8] gives an overview of the results of numerical simulation of reflectors of parabolic antennas of various designs. The authors indicate difficulties of maintaining a geometric shape of the reflecting surface while opening the frame of an antenna of large size. They point to the possibility to construct antennas in orbit. In paper [9], a geometrical model is described that includes information about the shape and size of a large-scale rod structure that can unfold. In order to build equations of motion, a variety of techniques can be applied with some of them outlined in [10], which specifies certain features in the estimation of the unfolding of cosmic rod structures.

In order to apply woven wire cloth at high frequencies in orbit, it is required to consider durability, elasticity, brittle resistance, surface electrical resistance, maximal isotropy of mechanical and radio-physical properties. With a decrease in the size of cells in a woven wire cloth, the specified indicators are difficult to maintain while unfolding an antenna. In addition [11], in order to reduce contact resistance, the wires should be covered with gold and other materials. This significantly increases the cost of an antenna, delivered folded, compared to that woven in orbit. It is emphasized that the structure of the woven wire cloth must be chosen according to the predefined parameters of electromagnetic wave.

When designing woven wire cloth for antennas for low frequencies (that is, ultralong waves), it is acceptable to create wire-cloth structures with large mesh cells. This points to the possibility of making such a woven wire cloth directly in orbit. The relevance of research into this area is indicated by that the use of space antennas for the ultralong wave range is appropriate for solving many applied tasks. These include: monitoring terrestrial plant covers, tasks related to hydrology, oceanography, geology, glaciology [12].

This explains why all “cosmic” states in the world pay special attention to this area. Construction of giant space objects directly in the orbit is the aim of planned innovative project SpiderFab, which is funded by NASA [13]. Pilot version should be ready by 2020. Automated robots of the project SpiderFab will be shaped as spiders that would use cosmic 3D printers to weave a “web” of carbon fiber delivered from the Earth. With the growth of the “web”, a robot would move along it like real spiders do [14]. The “spider's web” would create the basis for constructing space antennas, mirrors, and other infrastructure parts of spacecraft. Possible physical appearance of the device is shown in Fig. 1.

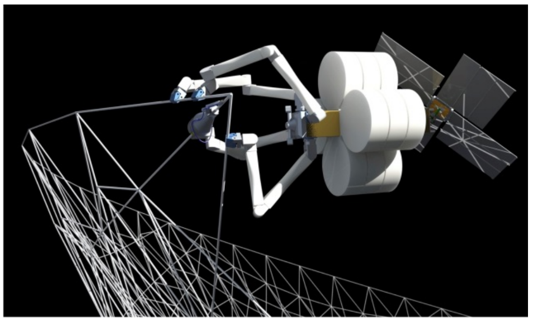


Fig. 1. Technological layout of SpiderFab [15]

The implementation of these plans should be responsibility of the Washington-based high-tech company Tethers Unlimited Inc. Original materials in the form of coils of fibers or blocks of polymers are to be fabricated on Earth, and then to be delivered by missile carriers to assemble in orbit. Automated robots of SpiderFab will use the materials to create giant structures whose designs should be optimized for application in space, rather than add load during a rocket launch.

SpiderFab will use the robot Trusselator for space structures, which would combine features of a 3D printer and a knitting machine [16]. One side of the cylindrical body will hold a coil with a thread (for example, a carbon fiber, or other polymer). The second side of the cylindrical body will hold an extruder to fabricate the pipes for would-be rod structures. In the end, the Trusselator apparatus, with a length of about a meter, would be able to assemble a rod structure with length in tens of meters. It is assumed that the robot Trusselator, by using a manipulator, will be capable of connecting original rod structures into larger complex structures, covering them with solar panels, light or radio-reflecting film, performing other operations depending on the goals of the mission.

Paper [17, 18] describe a technique for unfolding in weightlessness a rod structure, which is the analogue of a multilink pendulum. This idea of using pendulum oscillations in weightlessness can be employed for the creation of a model to weave a wire cloth. For this purpose, oscillations of the pendulum system must be aligned so that while moving in space trajectories of loads would take the form of zigzag lines, which would by a geometrical shape define the elements of a wire cloth. As an example, it is appropriate to choose a dual pendulum. Therefore, in order to create a model of weaving a woven wire cloth in weightlessness, we shall apply oscillations of a dual pendulum. It is planned to investigate only the geometrical model of a woven wire cloth formation. Paper [19] demonstrated the possibility of applying Lagrange equations of the second kind under condition of weightlessness (that is, in the absence of gravity force), and as a result, the "zero" potential energy of a mechanical system. These results are appropriate to utilize in practice for geometrical modeling of a dual pendulum oscillations in weightlessness, which would underlie the model of weaving a wire cloth in weightlessness.

Specifically, it is still an open scientific niche to explore a method for making a woven wire cloth in orbit using oscillations of a dual pendulum. This allowed us to define the direction of our research. By using the effect of inertial unfolding of a dual pendulum on the imaginary plane, we aim to construct a geometrical model of the process of weaving a wire cloth in weightlessness. In a given case, we shall consider that the movement of a dual pendulum is carried out through two jet engines installed in the nodal elements. In addition, the imaginary plane should move in parallel in space in the direction of its normal while the wire must come from the final nodal element of the pendulum. The existence of extensible links in the structure should not fundamentally affect the universal implementation of weaving a wire cloth in weightlessness.

3. The aim and objectives of the study

The aim of present study is to develop a geometrical model of the process of weaving in weightlessness a wire cloth using inertial unfolding of the system of dual pendulums on the imaginary plane. The imaginary plane must move in space in parallel in the direction of its normal while the wire must come from the final nodal of elements of the pendulums. The movement of each pendulum is enabled through two jet pulse engines installed in its nodal elements.

To accomplish the set aim, the following tasks have been solved:

– to construct and resolve a system of Lagrange differential equations of the second kind to describe flat oscillations in weightlessness of a dual pendulum on the platform;

– to develop a scheme of enabling oscillations of the pendulum by the impact of pulses from jet engines on each of its two nodal elements;

– to determine the values for initial conditions of oscillations in weightlessness of a dual pendulum, which would ensure the displacement of its second load along the platform;

– to design a scheme of spatial geometrical model of the process of weaving a wire cloth by oscillations of the system of dual pendulums on the imaginary plane (platform) when this plane moves in parallel while the wires come from the final nodal elements of the pendulums;

– to prepare test examples of the geometrical model of the process of weaving a wire cloth using inertial unfolding of the system of dual pendulums in weightlessness depending on parameters.

4. Development of geometrical model of the process of weaving a wire cloth using inertial unfolding of a dual pendulum

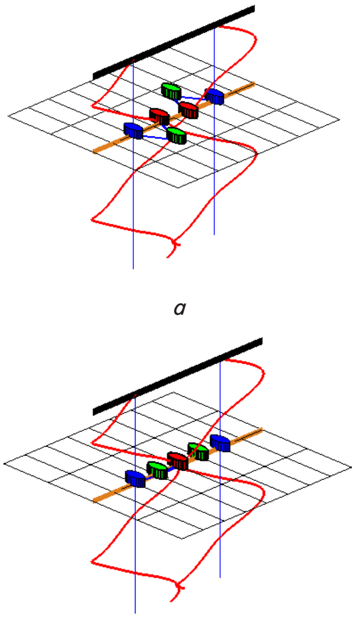
4. 1. Explanation of the general scheme of the process of weaving a wire cloth

Substantiation of the choice of design parameters of the process of weaving a wire cloth and confirmation of the feasibility of this process is related to conducting a detailed geometrical modeling using effective mathematical model.

In order to explain the idealized scheme of the process of weaving a wire cloth in weightlessness, we created an animated video, which can be found at the internet site [20]. Figure 2*a, b* shows framed excerpts from this video.

Assume that dual pendulums, located in line on a massive platform, perform oscillations within the imaginary plane. Fig. 2, *a, b* shows only two of them as an example. Fixed points of the pendulums (marked in blue) are attached to the solid platform, which, along with the imaginary plane, moves in parallel in the direction of its normal. Black line marks a "start" position of the platform; yellow line denotes current position in the process of displacement. As a result of maintaining the required values for the initial conditions, the oscillating process of a dual pendulum can be enabled so that its second load moves along the platform. Then the zigzag-like mesh elements in the process of formation (marked in red) will be created using the wires that come from the load loads of dual pendulums (Fig. 2). Starting position of the wires is fixed by another platform, located on the site of the "start" of the current platform. Green color denotes a load of ballast that can be used for technological needs.

Motors of each of the pendulums are a pair of pulse jet engines attached to the nodal elements. The same engines enable motion in weightlessness of the massive platform with pendulums. To create zigzags in a mesh, it is required that the last loads of each of the pendulums should move along the platform, with the distance between the points of attachment of pendulums allowing for the possibility of contact. Then, owing to the calculated ratios between parameters of oscillations of dual pendulums and speed of the platform displacement it is possible to obtain a process that would enable mesh formation in space. The zigzag-like mesh elements are fixed together by using point welding at the moments of contact between the last loads (Fig. 2, *b*). It is very important that prior to this the loads would move towards one another, and the moment of contact (in fact, an impact) should be used to trigger sensors for enabling the automated point welding.


a

b

Fig. 2. Displacements of dual pendulums: *a* – current position of loads; *b* – position at the moment of contact of last loads

The specified scheme of the process of weaving a wire cloth allows for any number of pendulums, located in line. A given method of weaving a wire cloth is not strict to its dimensions because its "width" is determined by the number of pendulums on the platform, and its "length" is defined by the wire available in the last loads of pendulums.

4. 2. Description of the oscillation of a dual pendulum in weightlessness by using Lagrange equations of the second kind

We shall fix under conditions of weightlessness an imaginary plane with the Cartesian coordinates *Oxy* and consider the idealized mathematical model of a dual pendulum on it. Assume that the pendulum consists of weightless non-stretchable rods of length d_1 and d_2, connected by hinges through nodal points, which have loads of masses m_1 and m_2 mounted onto them. Hinge movement should provide for the displacement of loads only within the chosen plane. We shall consider that the point of pendulum attachment is immobile in the plane coordinate system because it is connected to the platform whose mass is orders of magnitude larger than the mass of loads in the nodes. We mark with black color the fixed point of a dual pendulum in the plane coordinate system in all subsequent figures in the present article.

Let the beginning of the first link of the pendulum coincides with the origin of coordinates. Reference direction is the y the axis. The generalized coordinates are angles u(t) and v(t), created by respective links of the y axis direction y on the plane (Fig. 3).

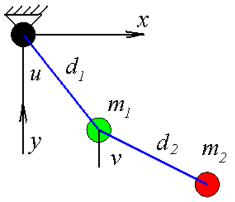


Fig. 3. Schematic of a dual pendulum

In order to determine mutual arrangement of the elements of a dual pendulum in time under condition of absence of dissipative forces, we shall apply [21] Lagrange equation of the second kind.

$$\frac{\partial}{\partial t}\left(\frac{\partial}{\partial u'}L\right)-\frac{\partial}{\partial u'}(L)=0,$$

$$\frac{\partial}{\partial t}\left(\frac{\partial}{\partial v'}L\right)-\frac{\partial}{\partial v'}(L)=0, \qquad (1)$$

where L is the lagrangian,

$$u\,'=\frac{d}{dt}u(t) \text{ and } v\,'=\frac{d}{dt}v(t)$$

are the derivatives from the functions of description of the generalized coordinate (that is, the value of the "initial" instantaneous velocity of the increase in the corresponding angle).

Using the generalized coordinates, we shall compute coordinates of the nodes of the pendulum:

$$x_1(t)=d_1\sin(u(t));\;\; y_1(t)=d_1\cos(u(t));$$

$$x_2(t)=x_1(t)+d_2\sin(v(t));$$

$$y_2(t)=y_1(t)+d_2\cos(v(t)); \qquad (2)$$

The lagrangian will be calculated by equating it to kinetic energy:

$$L=0{,}5\left[m_1\left(\dot{x}_1^2+\dot{y}_1^2\right)+m_2\left(\dot{x}_2^2+\dot{y}_2^2\right)\right]. \qquad (3)$$

After substituting formula (3) in expressions of equations (1), we shall obtain a system of two differential equations relative to functions u(t) and v(t), (not included here due to bulkiness). When solving it, one should take into consideration values of the initial angles of deviations $u(0)$ and $v(0)$, as well as values of the initial velocities $u'(0)$, $v'(0)$, applied to the deviation angles. Then the system of Lagrange equations of the second kind can be solved in the environment of the software package *maple* by approximate Runge–Kutta method; the resulting approximated solutions are to be denoted by characters $U(t)$, $V(t)$.

We shall determine coordinates of nodal points at time moment t on the plane of the *Oxy* coordinate system. To do this, we shall apply expressions (2) to calculate coordinates of the pendulum nodes using the generalized coordinates, replacing lowercase letters u and v in them with capital U

and *V*. When employing the *maple* software, in addition to the displacement of nodal points, one can determine velocities, which makes it possible to construct respective phase trajectories of displacement, thereby analyzing the dynamics of the process in general.

4. 3. Explanation of the idea of initiating oscillations in weightlessness

In order to ensure the required displacement of elements of a dual pendulum in weightlessness, it is necessary that two pulses of the two jet engines should impact its nodes. That is, initiating the oscillations of the pendulum will be carried out by choosing the coordinates of vector of initial velocities, applied to each of the two deviation angles. For example, $\{u'(0),\ v'(0)\}$ indicates that the first load with mass m_1 received the pulse of magnitude $m_1 u'(0)$, while the second load with weight m_2 was given the pulse of magnitude $m_2 v'(0)$). Vectors of velocity direction setting will be located perpendicular to the corresponding link of the pendulum in the final points (Fig. 4).

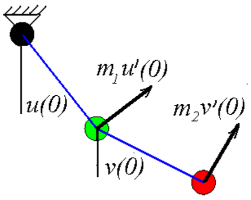


Fig. 4. Initiating the oscillations of dual pendulum

The chosen initiation of oscillations is the geometrical interpretation of action of the pulse jet engines. With respect to velocities $u'(0)$ and $v'(0)$, given to the nodal elements by jet engines, the pendulum system must subsequently unfold by inertia. This explains the term "inertial system of unfolding." Thus, the magnitudes of pulses form the first and the second jet engines can be related to the values of $u'(0)$ and $v'(0)$, respectively. Note that in the folded position the initial position of a possibly massive jet engine is in the region of pendulum attachment to the platform, which is convenient for transportation.

5. Computer simulation of the unfolding of a dual pendulum

5. 1. Determining the required values for initial velocities of deviation angles of a dual pendulum.

A two-link frame structure of the pendulum has to be delivered into orbit folded. Then the vector of values of initial deviation angles in a trivial case will accept coordinates $\{0,\ \pi\}$. The unfolding of a dual pendulum will depend on the values of length of links d_1 and d_2; masses of loads m_1 and m_2; initial positions of links $u(0)=0$ and $v(0)=\pi$, as well as initial velocities $u'(0)$ and $v'(0)$ of the unfolding of angles. The most practical when choosing the parameters will be initial velocities $u'(0)$ and $v'(0)$. Given such a change, there is no need to make any adjustments to the design of the pendulum, it is only required to provide for the magnitude of pulse from a jet engine. We shall herewith denote all values of parameters in conditional magnitudes because we shall consider only a geometrical model of the formation of a woven wire cloth (rather than the actual design of the device).

It is theoretically interesting to obtain results related to the calculation of values for initial velocities of the unfolding of the angles, which would ensure a non-chaotic trajectory of motion of the last load. We shall choose the following values od parameters as an example: $d_1=0.5$; $d_2=1$; $m_1=100$; $m_2=1$; $u(0)=0$; $v(0)=\pi$; $u'(0)=1$; $v'(0)=0.417$. The magnitudes of these parameters allow for the existence of the non-chaotic trajectory of the second load of the pendulum on the imaginary plane in weightlessness (Fig. 5)

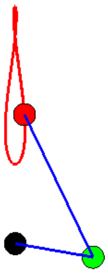


Fig. 5. Example of a non-chaotic trajectory of the second load of the pendulum on the imaginary plane in weightlessness

We shall choose the following parameters for the second example: $d_1=1$; $d_2=1$; $m_1=1$; $m_2=1$; $u(0)=0$; $v(0)=\pi$; $u'(0)=1$; $v'(0)=-0.65$. These are the values for the initial velocities of deviation angles that can enable approximate displacement of the second load along the platform. Fig. 6 shows the trajectory of displacement of the second load under condition that the platform is horizontal.

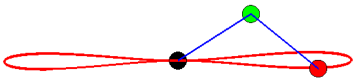


Fig. 6. Trajectory of displacement of the last load in weightlessness under condition $u'(0)=1$; $v'(0)=-0.65$

For comparison, note that the same parameters, but in the field of gravity, will produce the form of oscillations of the dual pendulum that is shown in Fig. 7

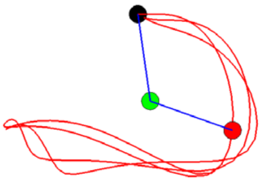


Fig. 7. Trajectory of displacement of the last load in the field of gravity

Consequently, it is possible to influence the unfolding of a dual pendulum applying the values of initial velocities $u'(0)$ and $v'(0)$ of the nodal elements, which are advisable to be related to the magnitudes of pulses of the jet engines.

5. 2. Test examples of geometrical modeling of weaving a wire cloth

As a result of execution of the constructed program for selected time moment t, we shall obtain approximated values for the current magnitudes of angles $u(t)$, $v(t)$, as well as their derivatives. This information will be employed when building phase trajectories of coordinate functions. This information could be applied for further calculations related to the dynamics and strength of structures.

Example 1. Links of the dual pendulum are the same; parameters accept the following values: d_1=1; d_2=1; m_1=1; m_2=1; $u(0)=0$; $v(0)=\pi$; $u'(0)=1$; $v'(0)=-0.65$.

Fig. 6 shows the trajectory of displacement of the last load in weightlessness for the given parameters (the main condition being $u'(0)=1$; $v'(0)=-0.65$. Fig. 8, 9 show phase trajectories of coordinate functions from example 1.

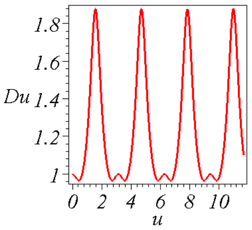


Fig. 8. Phase trajectory of coordinate $u(t)$ from example 1

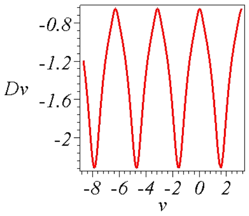


Fig. 9. Phase trajectory of coordinate $v(t)$ from example 1

The dependences obtained for $u(t)$ and $v(t)$ allow us to perform geometrical modeling of the technique for weaving a wire cloth using the unfolding of a two-link pendulum. The developed software demonstrates a sequence of N frames of animated images depending on the time of the unfolding of a set of dual pendulums, located on the platform [20]. Fig. 10 shows axonometric image of the motion phase of one pendulum in the process of unfolding. Zigzag of the antenna is obtained as the trace of displacement of the last load of the pendulum in space (marked in red). It is believed that the wire comes from the last load at a rate aligned with the speed of platform displacement.

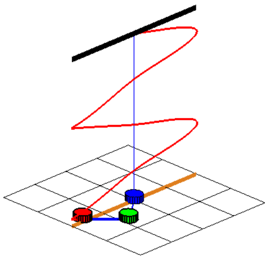


Fig. 10. Motion phase of the unfolding of a dual pendulum from example 1

In order to fix the zigzags in space, it is necessary to employ the unfolding of several dual pendulums. In this case, neighboring pendulums must unfold in counter phase. Fig. 11 shows one of the phases of unfolding three dual pendulums. Automated point welding of the wires is performed at the moments of contacts between the last loads of pendulums.

The modified scheme for weaving a wire cloth employs the unfolding of two dual pendulums with a common point of attachment. Fig. 12 shows one of the unfolding phases of two dual pendulums with a common point of attachment.

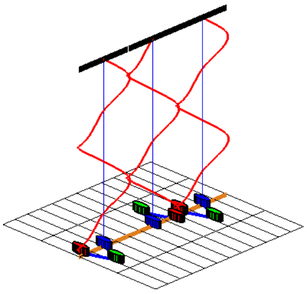


Fig. 11. Motion phase of the unfolding of three dual pendulums from example 1

Fig. 13 shows motion phases of the unfolding of four dual pendulums with a common point of attachment.

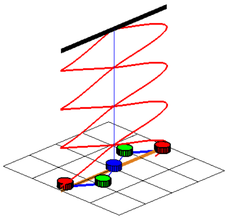


Fig. 12. Motion phase of the unfolding of two dual pendulums with a common point of attachment from example 1

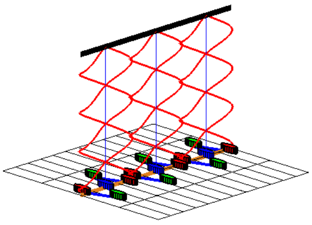


Fig. 13. Motion phase of the unfolding of four dual pendulums with a common point of attachment from example 1

Example 2. The second link of the dual pendulum is twice less in length than the first link. Parameters accept the following values: d_1=1; d_2=0.5; m_1=1; m_2=1; $u(0)=0$; $v(0)=\pi$; $u'(0)=1$; $v'(0)=-0.045$.

Fig. 14 shows motion trajectory of displacement of the last load in weightlessness for these parameters (the main condition being $u'(0)=1$; $v'(0)=-0.045$. Fig. 15, 16 show phase trajectories of coordinate functions from example 2.

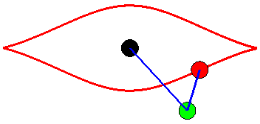


Fig. 14. Trajectory of displacement of the last load in weightlessness under condition $u'(0)=1$; $v'(0)=-0.045$

For a given and subsequent examples, we shall refer to respective comments in the first example. Fig. 17 shows axonometric images of the motion phase of one pendulum in the process of unfolding. Fig. 18–20 show combined pendulums.

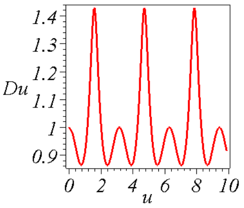


Fig. 15. Phase trajectory of coordinate *u*(t) from example 2

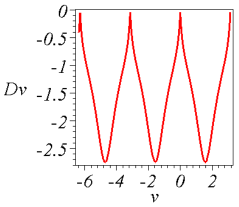


Fig. 16. Phase trajectory of coordinate *v*(t) from example 2

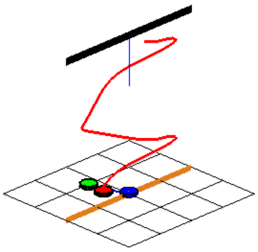


Fig. 17. Motion phase of the unfolding of a dual pendulum from example 2

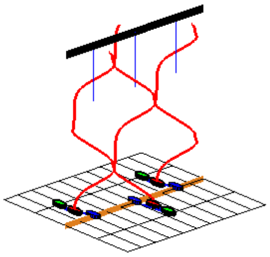


Fig. 18. Motion phase of the unfolding of three dual pendulums from example 2

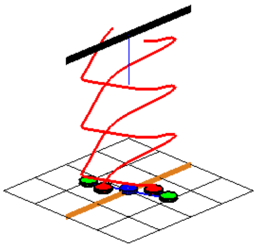


Fig. 19. Motion phase of the unfolding of two dual pendulums with a common point of attachment from example 2

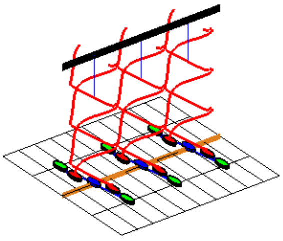


Fig. 20. Motion phase of the unfolding of four dual pendulums with a common point of attachment from example 2

Example 3. The first link of the dual pendulum is less than half of the length of the second link. Parameters accept the following values: d_1=0.5; d_2=1; m_1=1; m_2=1; $u(0)=0$; $v(0)=\pi$; $u'(0)=1$; $v'(0)=-1.54$.

Fig. 21 shows trajectory of displacement of the last load in weightlessness for these parameters (the main condition being $u'(0)=1$; $v'(0)=-1.54$. Fig. 22, 23 show respective phase trajectories of coordinate functions from example 3.

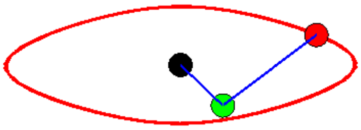


Fig. 21. Trajectory of displacement of the last load in weightlessness under condition $u'(0)=1$; $v'(0)=-1.54$

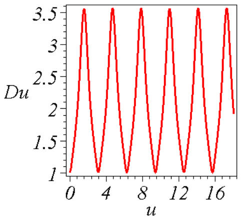


Fig. 22. Phase trajectory of coordinate *u*(t) from example 3

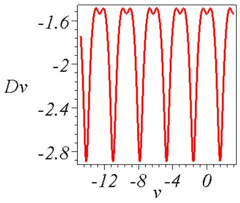


Fig. 23. Phase trajectory of coordinate *v*(t) from example 3

Fig. 24–27 show motion phases of the unfolding of a dual pendulum and its modifications.

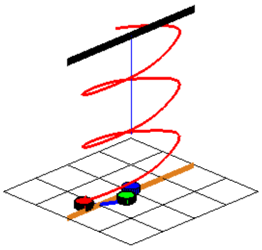


Fig. 24. Motion phase of the unfolding of a dual pendulum from example 3

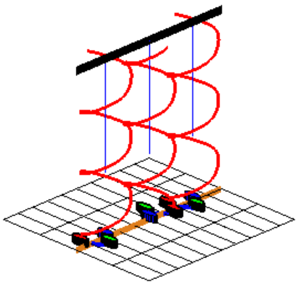


Fig. 25. Motion phase of the unfolding of three dual pendulums from example 3

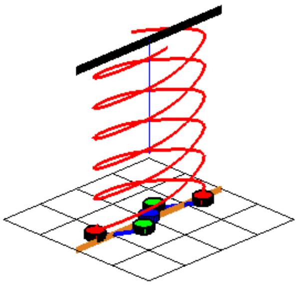


Fig. 26. Motion phase of the unfolding of two dual pendulums with a common point of attachment from example 3

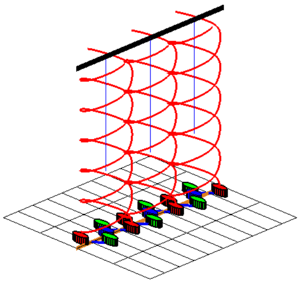


Fig. 27. Motion phase of the unfolding of four dual pendulums with a common point of attachment from example 3

Example 4. Links of the dual pendulum are the same. The mass of the second load is two orders of magnitude larger than the mass of the first load. Parameters accept the following values: $d_1=1$; $d_2=1$; $m_1=1$; $m_2=100$; $u(0)=0$; $v(0)=\pi$; $u'(0)=1$; $v'(0)=-0.9$.

Fig. 28 shows trajectory of displacement of the last load in weightlessness for these parameters (the main condition being $u'(0)=1$; $v'(0)=-0.9$. Fig. 29, 30 show phase trajectories of coordinate functions from example 3.

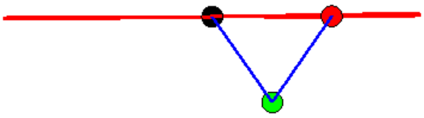


Fig. 28. Trajectory of displacement of the last load in weightlessness under condition $u'(0)=1$; $v'(0)=-0.9$

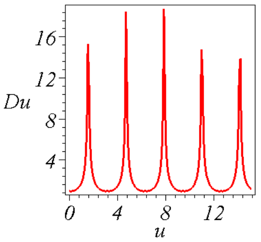


Fig. 29. Phase trajectory of coordinate *u*(t) from example 4

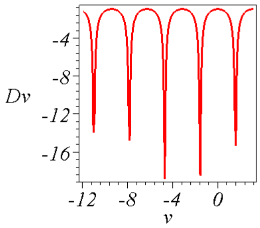


Fig. 30. Phase trajectory of coordinate *v*(t) from example 4

Fig. 31–34 show axonometric images of the motion phase of pendulums in the process of unfolding.

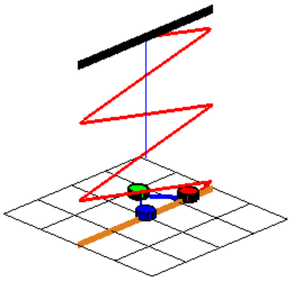


Fig. 31. Motion phase of the unfolding of a dual pendulum from example 4

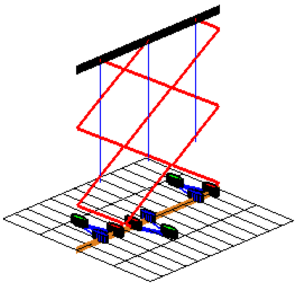


Fig. 32. Motion phase of the unfolding of three dual pendulums from example 4

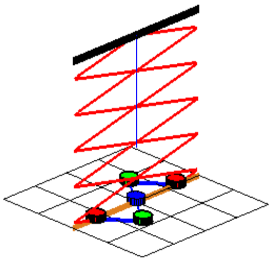


Fig. 33. Motion phase of the unfolding of two dual pendulums with a common point of attachment from example 4

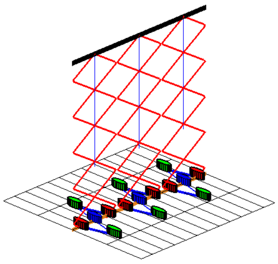


Fig. 34. Motion phase of the unfolding of four dual pendulums with a common point of attachment from example 4

When comparing results shown in the examples, it should be noted that the most acceptable variant of the pendulum is the one with links of the same length as the motion speed of the second load will be less compared to other variants. This should exert a positive effect on the dynamics of the structure. Although a woven wire cloth with better shapes of cells was obtained in example four, where motion speed of the second load would be the largest.

Summing up the above considerations, it should be noted that there may be other variants of the implementation of a model for weaving a wire cloth using inertial unfolding of a dual pendulum on the imaginary plane. The result of the research undertaken is the construction of a geometrical model for creating in weightlessness a spatial shape of cells in a wire cloth. The developed idealized geometrical model for the fabrication of woven wire cloth for space antennas is designed to illustrate the main stages in the calculation of such massive structures.

6. Discussion of results of examining a geometric model for weaving a wire cloth using the inertial unfolding of a dual pendulum

The results obtained can be explained by the possibility to apply the Lagrange variational principle to the estimation of mechanical structures with respect to kinematic relations and employing "zero" potential energy of the mechanical system. This allowed us to use the Lagrange equations of the second kind to describe the motion of dual pendulums in weightlessness.

It is clear that a given geometrical model for the formation of elements of a woven wire cloth in weightlessness requires further research to bring it closer to the actual structure. First, it is necessary to take into consideration the moments of inertia at rotation of the pendulum elements. Second, it is necessary to take into consideration attenuation of oscillations of a dual pendulum over time, and reduction in the mass of the last load due to the consumption of wire. It would be interesting to study the effect of reduction in the mass of the last load on the parameters of oscillation attenuation. And, third, a simultaneous actuation is required of the jet engines installed on the loads of pendulums and on the platform. They would ensure the action of pulses of the estimated magnitudes in order to "accelerate" the pendulums.

The development of a given research field would imply development of a radial technique for weaving a wire cloth, as well as the method for weaving a wire cloth in the form of a "stocking". Of interest as well is to explore, instead of inertial unfolding of two-link pendulums, a possibility of employing electric motors to rotate the links of pendulums. This direction might be feasible under condition of energy supply to the platform with pendulums.

The difficulties of research development in this direction are associated mainly with the welding technique. There are obvious differences between space conditions and those on Earth – first of all, weightlessness and deep vacuum at almost unlimited rate of gas diffusion from a welding region. All this necessitates further research.

The benefit of the proposed method of weaving a wire cloth in weightlessness is as follows:

– a procedure for weaving a wire cloth using the inertial unfolding of a system of dual pendulums would solve the problem of creating large woven wire cloth in orbit; their "width" would be determined by the number of pendulums on the platform while their "length" – by the reserve of wire in the last loads of pendulums;

– counter movements and moments of touch (as impacts) of the last loads of pendulums could be used to trigger the sensors that would enable automated point welding of compatible zigzag-like elements of the mesh;

– owing to the use of pairwise connected dual pendulums with a common point of attachment and oscillations of links in counter phase, it is possible to attempt to weaken (or even eliminate) rotating moments of the structure in general;

– for a dual pendulum, the starting position of a jet engine is in the region of its attachment to the platform, which is convenient for delivery and installation of the structure.

7. Conclusions

We constructed and solved the system of Lagrange differential equations of the second kind that describe flat oscillations of the dual pendulum in weightlessness, which made it possible to determine its parameters to enable non-chaotic (periodic) oscillations.

A circuit is developed to initiate oscillations of the pendulum by influencing, with the pulses of jet engines, each of its two nodal elements, which allowed us to significantly simplify design of the motion means compared, for example, to the use of step electric motors.

We determined the values for initial conditions of oscillations in weightlessness of the variants of a dual pendulum, which would ensure displacement of its second load along the mobile platform, which made it possible to form zigzag-like elements of a wire cloth from wire.

A circuit is developed of a spatial geometrical model of weaving a wire cloth by oscillating a system of dual pendulums on the imaginary plane (platform) when this plane moves in parallel while the wires come from the last nodal elements of pendulums, which made it possible to create a woven wire cloth with various cell patterns depending on the parameters of dual pendulums.

The given test examples of the geometrical model for the process of weaving a wire cloth using the inertial unfolding of a system of dual pendulums in weightlessness explain the possibility of visualization of the technique for creating a woven wire cloth in the case of its practical implementation.

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