3-D Radome-Enclosed Aperture Antenna Analyses and Far-Side Radiation

Ilya Vladimirovich Sukharevsky, Sergey V. Vazhinsky, and Ilya O. Sukharevsky, *Student Member, IEEE*

***Abstract*—Physical optics integral representations of the fields are given for the 3-D model of the aperture antenna with specified ampliphase excitation law enclosed in a radome. The problem is reduced to finding fields of a plane wave diffracted on the "symmetrized" radome. This model is used for calculations of radiation patterns and for analyses of far-field radiation. Peculiarities of the ray pattern and caustics are analyzed using geometrical optics (geometrical theory of diffraction) method. The contribution of the stationary phase points in the aperture to the far-side radiation has been investigated. Results of numerical calculations are presented.**

***Index Terms*—Aperture antenna, geometrical optics, geometrical theory of diffraction, physical optics, radomes, sidelobe supression.**

I. Introduction

RADOMES are an inevitable component of antenna systems and radars, both airborne and ground-based. Reflections of an antenna field from the inner walls of the radome may cause significant pattern deformations and increase sidelobe levels. Although various integral equation techniques and FDTD methods [1]–[4] have been employed for radome simulation during the last decade, asymptotic approaches (see, for example, [5]–[10]) are still more suitable for 3-D modeling of electrically large radomes due to low computational costs.

Aperture antennas in an inhomogeneous medium have been studied for various applications. In [11] the aperture antenna located below a linearly inhomogeneous semispace was studied in the context of hydroacoustic or troposphere wave propagation. The method proposed in [12] adopted a ray tracing technique to obtain a projected image of source distribution.

Most of aperture antenna models (considered with respect to radomes, for example in [5]–[7]) can be interpreted as a flat aperture with a perfectly electrically conducting flange. However, this approximation provides valid results only for the main lobe and first sidelobes of directivity pattern. In contrast with this approach, another useful interpretation of aperture antenna as a hole in an ideally black screen was proposed in [13]. In this case, the incident field is absorbed by flanges; hence, the reflection from "fictitious" flanges does not contribute to the far-side radiation of the antenna system.

Here we employed a mathematically strict model of the radome-enclosed aperture antenna [14] based on the generalized principle of mirror images. It allows us to generalize the method of equivalent currents for aperture antenna calculations [13] in cases when the aperture antenna radiates to the semispace in the presence of arbitrary scatterers (dielectric, conducting or magnetic bodies). Simple, exact and approximate formulae have a clear physical interpretation and allow to calculate correctly directivity pattern in the whole semispace. The problem of antenna radiation is reduced to the diffraction of a plane wave on the "symmetrized" radome (the similar "symmetrized" model was used in [15] for simulating of a radome-enclosed dipole array backed by a ground plane in 2-D, and in [16] for representations of the fields of an aperture antenna enclosed in a spherical chiral radome).

The electromagnetic wave incidence can be calculated with a conventional geometrical optics (GO) method or geometrical theory of diffraction (GTD) algorithm. The first GTD consideration of diffraction on the layer was made by J.B. Keller [17]. Keller supposed the layer to be homogeneous and equidistant and described the field via integral equations derived easily from Green's formulae. Fixing k_0 (the wave number in a free space) and expanding all functions into series of δ (layer thickness), Keller obtained the first term of a diffracted field expansion and evaluated it with the stationary phase method for big k_0. However, results obtained via this method are valid only when $k_0\delta \gg 1$. The correct asymptotic consideration can be held only with respect to a prior relations with respect to $k_0\delta$. The method based on a combination of ray techniques and boundary-layer expansions allowed to solve this 3-D problem of diffraction on a thin layer in cases $k_0\delta \sim 1$ and $k_0\delta \ll 1$ [18], [19], and $k_0\delta \gg 1$ [20]. Heuristic assumption about the primary field representations in [18]–[20] was justified by the exact Green's function asymptotics of a point source in the presence of a thin layer obtained in [21]. In [22] this method was applied for the purpose of acoustic diffraction. It should be noted, that the first term of the expansion of the diffracted plane wave coincides with GO solution on the flat layer, and the next terms of expansion give corrections on curvature, non-equidistance, etc. These GTD methods consider slightly curved layers. However, diffraction from a tightly curved tip can be taken into account, as shown in [23].

Here the wave passage through the radome layer is calculated with the same accuracy as in [5]. However, the caustic influence on the fields reflected from the radome is taken into account.

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I. V. Sukharevsky (retired) was with the Military Academy of Air Defence, Kharkov 61077, Ukraine (e-mail: i_sukharevsky@rambler.ru).

S. E. Vazhinsky is with the National Academy of Defence of Ukraine, Kiev 03048, Ukraine (e-mail: sevzhasmin@mail.ru).

I. O. Sukharevsky is with the Institute for Radiophysics and Electronics of National Academy of Sciences of Ukraine, Kharkov 61085, Ukraine (e-mail: i.sukharevsky@gmail.com).

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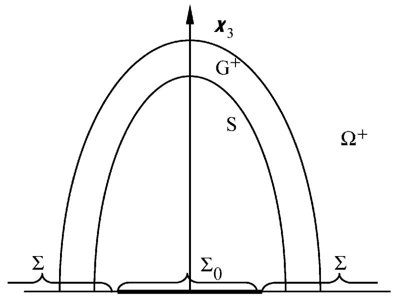


Fig. 1. Problem geometry. Aperture antenna Σ_0 enclosed in radome G^+.

Using the developed hybrid PO/GO (PO/GTD) algorithm we show that radar-induced distortions of the far-field pattern are mainly caused by the reflections from a small area of the radome wall associated with a stationary phase point in the aperture. This may open crucial opportunities to suppress far sidelobes (for example, by disconnecting corresponding elements of antenna array). Detection of stationary phase points can be also used for asymptotic approximation of the field by the method of stationary phase in 3-D, as it is done in [6] in 2-D case.

Sections II is devoted to the integral representation of a field radiated by the discussed system. The ray pattern and caustics are analyzed by GO (GTD) techniques in Section III. The detection of the stationary phase points (SPPs) is analyzed in Section IV. The results of 3-D modeling of a circular aperture antenna enclosed with a parabolic radome in support of our concept about the significant SPP influence on the far-side radiation field are presented in Section V.

II. Integral Representations of the Fields

Introduce Cartesian coordinates (x_1, x_2, x_3). Let aperture Σ_0 is in the plane $x_3 = 0$ and radiates to the semi-space Ω^+ $(x_3 > 0)$ (Fig. 1). Radiated field $\vec{E}(\vec{x})$, $\vec{H}(\vec{x})$ is induced by some sources in semi-space Ω^- $(x_3 < 0)$. Domain Ω^+ contains dielectric radome G^+ with permittivities $\varepsilon(\vec{x})$, $\mu(\vec{x})$ (generally, variable). Consider two assumptions about the physical properties of surface Σ:

A—Σ is the ideally conducting surface $(\vec{E}_t|_\Sigma = 0)$;

B—Σ is the ideally magnetic surface $(\vec{H}_t|_\Sigma = 0)$.

Denote $(\vec{E}^A, \vec{H}^A)$ and $(\vec{E}^B, \vec{H}^B)$ for the fields induced by aperture in domain Ω^+ in problems A and B. Consider also their half sums

$$\begin{cases} \vec{E}^C = \frac{1}{2}(\vec{E}^A + \vec{E}^B) \\ \vec{H}^C = \frac{1}{2}(\vec{H}^A + \vec{H}^B). \end{cases} \quad (1)$$

"Averaged" field (1) can be interpreted as a field corresponded to Macdonald's model [24] of ideally black surface Σ. Using Lorentz lemma and generalized principle of mirror images [14], derive for any point $\vec{x}_0 \in \Omega^+$ and any vector of receiving polarization $\vec{p}$

$$j\omega\vec{p}\cdot\vec{E}^C(\vec{x}_0) = \int_{\Sigma_0} \Big(\vec{E}^A(\vec{x}) \times \vec{H}_1(\vec{x}|\vec{x}_0, \vec{p}) - \vec{H}^B(\vec{x}) \times \vec{E}_1(\vec{x}|\vec{x}_0, \vec{p})\Big)\, d\vec{S} \quad (2)$$

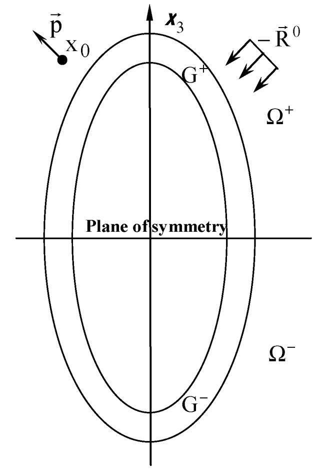


Fig. 2. "Symmetrized" radome.

where $d\vec{S} = \vec{n}dS$, $\vec{n}$ is the unit vector, normal to Σ_0 and directed to Ω^-; $\vec{E}_1, \vec{H}_1$ is electromagnetic field induced by a point source (electric dipole) in a space containing a closed dielectric shell, which is symmetrical with respect to plane $x_3 = 0$ (Fig. 2).

From exact formulae (2) go to the approximation of physical optics. When edge effects in the aperture are small

$$\vec{E}^A(\vec{x}) \approx \vec{E}^B(\vec{x})$$
$$\vec{H}^A(\vec{x}) \approx \vec{H}^B(\vec{x}).$$

Then, omitting indices A, B, we get

$$j\omega\vec{p}\cdot\vec{E}^C(\vec{x}_0) = \int_{\Sigma_0} \Big(\vec{E}(\vec{x}) \times \vec{H}_1(\vec{x}|\vec{x}_0, \vec{p}) - \vec{H}(\vec{x}) \times \vec{E}_1(\vec{x}|\vec{x}_0, \vec{p})\Big)\, d\vec{S}. \quad (3)$$

Right-hand integral (3) expresses field of the radiating aperture through the given distributions of $\vec{E}$, $\vec{H}$ in the aperture, and the field, diffracted on the "symmetrized" radome. Equation (3) means that in the inhomogeneous medium the field, calculated by the method of equivalent currents, coincides (in PO approximation) with "averaged" field of Macdonald model. Thus, these formulae generalize results of [13] for the case of inhomogeneous medium.

From (2) and (3), obtain formulae for complex directivity pattern of radiating system $\vec{E}(\vec{R}_0)$, where $\vec{R}_0$ is a unit vector of an observation point in far-zone

$$j\omega\vec{p}\cdot\vec{E}^C(\vec{R}_0) = \int_{\Sigma_0} \Big(\vec{E}^A(\vec{x}) \times \vec{H}_1(\vec{x}, \vec{R}_0, \vec{p}) - \vec{H}^B(\vec{x}) \times \vec{E}_1(\vec{x}, \vec{R}_0, \vec{p})\Big)\, d\vec{S}.$$

— Exact formulae

$$j\omega\vec{p}\cdot\vec{E}^C(\vec{R}_0) = \int_{\Sigma_0} \Big(\vec{E}(\vec{x}) \times \vec{H}_1(\vec{x}, \vec{R}_0, \vec{p}) - \vec{H}(x) \times \vec{E}_1(\vec{x}, \vec{R}_0, \vec{p})\Big)\, d\vec{S} \quad (4)$$

— Approximate PO formula; $\vec{E}, \vec{H}$ is aperture distribution of tangential field in Kirchhof's approximation; $\vec{E}_1$, $\vec{H}_1$ is a

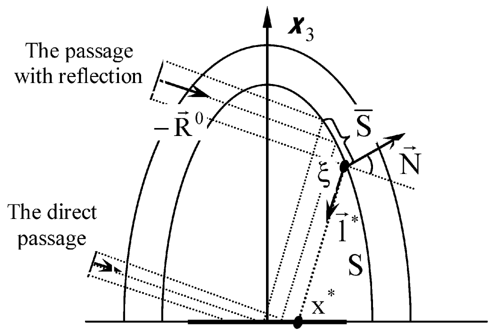


Fig. 3. Ray pattern.

field of a plane wave propagating in the direction $-\vec{R}_0$, which passed through the radome

$$\vec{E}_0 = \left(\vec{R}^0 \times (\vec{p} \times \vec{R}^0)\right) \exp\left(-ik_0(\vec{R}^0 \cdot \vec{x})\right)$$
$$\vec{H}_0 = (\vec{p} \times \vec{R}^0)\sqrt{\frac{\varepsilon_0}{\mu_0}} \exp\left(-ik_0(\vec{R}^0 \cdot \vec{x})\right)$$

where ε_0, μ_0, k_0 are permittivities and a wavenumber in free space, respectively.

Assume $\vec{H}_1(x, \vec{R}^0, \vec{p})$, $\vec{E}_1(x, \vec{R}^0, \vec{p})$ as a sum of the field reflected from the inner wall S of the radome and the field passed through the radome directly to the aperture (Fig. 3). Multi-reflections provide scattered field corrections of a higher order, so can be neglected due to electrically large sizes of the radome.

III. Plane Wave Passage Through the Radome Wall

For simplicity of presentation, let layer G be equidistant. Take into consideration curvilinear coordinates (σ_1, σ_2) counted along the lines of curvature in the point of reflection (ξ_1, ξ_2, ξ_3) on the internal surface S of the radome.

Represent the incident wave, diffracted wave on the internal surface of the radome, and the reflected wave, respectively, as

$$\vec{E}^0 \sim e^{-ik_0(\vec{R}_0 \cdot \vec{x})} \cdot \vec{E}_0^0(\vec{x}) \tag{5}$$
$$\vec{E} \sim e^{-ik_0(\vec{R}_0 \cdot \vec{\xi})} \cdot \vec{E}_0^0(\sigma_1, \sigma_2) \tag{6}$$

(here $\vec{\xi} = (\sigma_1, \sigma_2)$ is the point of the reflection)

$$\vec{E}^* \sim e^{ik_0\left(-\vec{R}_0 \cdot \vec{\xi} + |\vec{x} - \vec{\xi}|\right)} \cdot \vec{E}_0^*(\vec{x}). \tag{7}$$

In (7) $\vec{x} = \vec{\xi} + \vec{l}^* \cdot t$, where

$$\vec{l}^* = -\vec{R}^0 + 2\vec{N}(\vec{R}^0 \cdot \vec{N}), \quad \vec{N} = \vec{N}(\vec{\xi}), \quad t \geq 0 \tag{8}$$

where $\vec{\mathrm{N}}$ is the normal unit vector; see Fig. 3.

Phases of each of the fields (5)–(7) are equal to the same value $(\vec{R}^0 \cdot \vec{\xi})$ in a point $\vec{x} = \vec{\xi}$.

Vectors $\vec{E}_0^0$, $\vec{E}_0^*$ are GO vector amplitudes or the first terms of asymptotic expansion [18] ($k_0\delta \sim 1$; $\kappa_0\delta \ll 1$, $\kappa_0/k_0 \ll 1$, where κ_0 is the largest curvature of a radome surface and a wave-front).

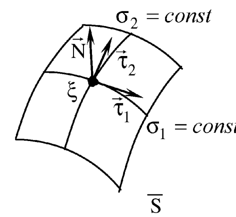


Fig. 4. Local coordinates.

The reflected field along the beam $\vec{x} = \vec{\xi} + \vec{l}^*(\vec{\xi}) \cdot t$ $(t \geq 0)$ represent via the well-known expression [25]

$$\begin{Bmatrix} \vec{E}_0^*(\vec{x}) \\ \vec{H}_0^*(\vec{x}) \end{Bmatrix} = \begin{Bmatrix} \vec{E}_0^*(\vec{\xi}) \\ \vec{H}_0^*(\vec{\xi}) \end{Bmatrix} \cdot \sqrt{\frac{D(0)}{D(t)}} \cdot e^{ik_0\left(-\vec{R}^0 \cdot \vec{\xi} + |\vec{x} - \vec{\xi}|\right)} \tag{9}$$

where $D(t) = \partial(x_1, x_2, x_3)/\partial(\sigma_1, \sigma_2, t)$ is the Jacobian of transition from beam coordinates to Cartesian ones.

A. The Development of $D(t)$. Caustic Surfaces

Let S be a fairly smooth strictly concave surface. Assume that $\overline{S}$ is a part of the surface S, where the normal unit $\vec{N} = \vec{N}(\vec{\xi})$ forms an acute angle with the given unit vector $-\vec{R}^0$ (Fig. 3). Let κ_1, κ_2 ($\kappa_1 \geq 0$; $\kappa_2 \geq 0$) be the main curvatures in point of reflection $\vec{\xi} = (\sigma_1, \sigma_2)$, and $\vec{\tau}_1$, $\vec{\tau}_2$ are the corresponding unit vectors (Fig. 4).

As we have described

$$\frac{\partial \vec{N}}{\partial \sigma_i} = \kappa_i \vec{\tau}_i \tag{10}$$
$$-\vec{R}^0 = \vec{\tau}_1 \cos\alpha_1 + \tau_2 \cos\alpha_2 + \vec{N}\cos\beta \tag{11}$$

with $0 < \beta < \pi/2$ on the surface $\overline{S}$, $\alpha_1, \alpha_2, \beta$ are angles between vector $\vec{R}^0$ and vectors $\vec{\tau}_1, \tau_2, \vec{N}$, respectively.

The reflected beam can be given by equation

$$\vec{r} = \vec{\xi}(\sigma_1, \sigma_2) + t\vec{l}^*(\sigma_1, \sigma_2) = \vec{\xi}(\sigma_1, \sigma_2) + t\left[-\vec{R}^0 + 2\vec{N}(\vec{R}^0 \cdot \vec{N})\right].$$

Taking into account (10) and (11), we derive

$$\begin{aligned} \frac{\partial \vec{r}}{\partial t} &= -\vec{R}^0 - 2\vec{N}\cos\beta \\ &= \vec{\tau}_1 \cos\alpha_1 + \vec{\tau}_2 \cos\alpha_2 - \vec{N}\cos\beta \\ \frac{\partial \vec{r}}{\partial \sigma_i} &= \vec{\tau}_i(1 - 2t\kappa_i \cos\beta) - 2t\kappa_i \cos\alpha_i \cdot \vec{N} \\ & \quad (i = 1; 2), \quad \text{and so} \\ D(t) &= \left(\frac{\partial \vec{r}}{\partial t}, \frac{\partial \vec{r}}{\partial \sigma_1}, \frac{\partial \vec{r}}{\partial \sigma_2}\right) \\ &= -\left(\cos\beta - 2t\widetilde{\kappa} + 4t^2\kappa_1\kappa_2 \cos\beta\right) \end{aligned}$$

where $\widetilde{\kappa} = \kappa_1 \sin^2\alpha_2 + \kappa_2 \sin^2\alpha_1$.

In particular 2D case ($\kappa_2 \equiv 0$; $\sin^2\alpha_2 \equiv 1$) we have

$$D(t) = -(\cos\beta - 2t\kappa_1).$$

Thus, in formula (9)

$$\sqrt{\frac{D(0)}{D(t)}} = 1/\sqrt{1-2\frac{\widetilde{\kappa}}{\cos\beta}t+4\kappa_1\kappa_2 t^2}.$$

A discriminant of $D(0)/D(t)$ can be presented either as

$$\Delta = \frac{\widetilde{\kappa}^2}{\cos^2\beta} - 4\kappa_1\kappa_2 \tag{11a}$$

or

$$\Delta = \frac{1}{\cos^2\beta}\left[\left(\kappa_1\sin^2\alpha_2-\kappa_2\sin^2\alpha_1\right)^2 + 4\kappa_1\kappa_2\cos^2\alpha_1\cdot\cos^2\alpha_2\right] \geq 0. \tag{11b}$$

From (11b) follows that the roots of the considered polynomial are real:

$$t_{1,2} = \left(\frac{\widetilde{\kappa}}{\cos\beta}\pm\sqrt{\Delta}\right)/4\kappa_1\kappa = t_{1,2}(\sigma_1,\sigma_2)$$

but since (11a) shows that $\sqrt{\Delta} \leq (\widetilde{\kappa}/\cos\beta)$

$$0 < t_1(\sigma_1,\sigma_2) \leq t_2(\sigma_1,\sigma_2).$$

Finally, we get

$$\sqrt{\frac{D(0)}{D(t)}} = 1/\sqrt{\left(1-\frac{t}{t_1(\sigma_1,\sigma_2)}\right)\left(1-\frac{t}{t_2(\sigma_1,\sigma_2)}\right)}.$$

Hence, in the discussed 3D case caustic surface $(D(t)=0)$ is comprised of two connected components, described in the beam coordinates as $t=t_1(\sigma_1,\sigma_2)$, $t=t_2(\sigma_1,\sigma_2)$. Asymptotic approximation (9) is valid while moving along the reflected beam from $t=0$ to the value $t<t_1-\Delta t$ (where Δt is some fixed positive number), but loses its physical meaning nearby $t=t_1$, because the one-to-one correspondence between the Cartesian and beam coordinates is not fulfilled. However [26], after passing the critical value the beam approximation is valid again, but in the modified form: after touching a beam caustic, wave phase declines as a leap on $\pi/2$. The similar phenomenon is occurring in the transition through the critical value t_2.

B. Stationary Phase Points of the Reflected Field

Let $x_3=g(x_1,x_2)$ be the equation of the inner surface S of the dielectric radome, either homogeneous or stratified.

Suppose the incident wave field has a flat front with a phase $-(\vec{R}^0\cdot\vec{x})$ and $-\vec{R}^0\cdot\vec{N}>0$ on $\overline{S}\subset S$ (Fig. 3); so that in accordance to (7), the beam reflected from S in a point $\vec{\xi}(\xi_1,\xi_2,g(\xi_1,\xi_2))$ has the phase

$$\Phi_0(\vec{x},\vec{\xi}) = -(\vec{R}^0\cdot\vec{\xi})+|\vec{x}-\vec{\xi}| \quad \text{when} \quad \vec{x}\in\Sigma_0. \tag{12}$$

For generality, assume the presence of currents performing electrical scanning in the direction $\vec{q}$. The corresponding mathematical model is the additional term $(\vec{q}\cdot\vec{x})$, $(|\vec{q}|=1)$ in the expression for the phase function. Instead of (12) we have

$$\Phi(\vec{x},\vec{\xi}) = (\vec{q}\cdot\vec{x})-(\vec{R}^0\cdot\vec{\xi})+|\vec{x}-\vec{\xi}| \tag{13}$$

where $\vec{\xi}\in\overline{S}$, $\vec{x}\in\Sigma_0$.

From the balance $\vec{x}=\vec{\xi}+\vec{l}^*t$ it is easy to imply

$$\left.\begin{array}{c} x_1=\xi_1+g\frac{l_1^*}{l_3^*}=x_1(\xi_1,\xi_2)\\ x_2=\xi_2+g\frac{l_2^*}{l_3^*}=x_2(\xi_1,\xi_2)\\ x_3=0.\end{array}\right\}. \tag{14}$$

With very broad assumptions about the surface $\overline{S}$, system of functions (14) is locally reversible in the neighborhood of each regular point (ξ_1^0,ξ_2^0) $(\partial(x_1,x_2)/\partial(\xi_1,\xi_2)\neq 0)$ and inverse functions $\xi_1(\mathrm{x}_1,\mathrm{x}_2)$, $\xi_2(\mathrm{x}_1,\mathrm{x}_2)$ are differentiable in the corresponding point (x_1^0,x_2^0).

Go to the study of phase-function $\Phi(\vec{x},\vec{\xi})$ derivatives. In the vicinity of each regular point $\xi_1=\xi_1(x_1,x_2)$ and $\xi_2=\xi_2(x_1,x_2)$, so $\xi_3=g(\xi_1,\xi_2)=\xi_3(x_1,x_2)$. Therefore, vector $\vec{\xi}$ of a variable point on $\overline{\mathrm{S}}$ can be considered as a function of curvilinear coordinates x_1 and x_2: $\vec{\xi}=\vec{\xi}(x_1,x_2)$, and the derivatives $\partial\vec{\xi}/\partial x_1$, $\partial\vec{\xi}/\partial x_2$ are vectors tangential to $\overline{S}$.

Taking into account that

$$\Phi=\Phi(\vec{x},\vec{\xi})=\Phi\left(\vec{x},\vec{\xi}(x_1,x_2)\right)=\Phi[x_1,x_2]$$
$$\text{when} \quad \vec{x}\in\Sigma_0, \quad \vec{\xi}\in\overline{S}$$
$$\text{derive}: \quad \frac{\partial\Phi[x_1,x_2]}{\partial x_i}=\frac{\partial\Phi(\vec{x},\vec{\xi})}{\partial x_i}+\sum_{k=1}^{3}\frac{\partial\Phi(\vec{x},\vec{\xi})}{\partial\xi_k}\frac{\partial\xi_k}{\partial x_i}$$
$$=q_i+\frac{x_i-\xi_i}{|\vec{x}-\vec{\xi}|}+\left(\frac{\vec{\xi}-\vec{x}}{|\vec{\xi}-\vec{x}|}-\vec{R}^0\right)\cdot\frac{\partial\vec{\xi}}{\partial x_i}$$
$$\mathrm{i}=1;2. \tag{15}$$

In consequence of (8) and the apparent equality $(\vec{\mathrm{x}}-\vec{\xi})/|\vec{\mathrm{x}}-\vec{\xi}|=\vec{\mathrm{l}}^*$, obtain

$$\frac{\vec{\xi}-\vec{x}}{|\vec{x}-\vec{\xi}|}-\vec{R}^0=-(\vec{l}^*+\vec{R}^0)=-2\vec{N}(\vec{R}^0\cdot\vec{N})$$

and since $\vec{N}(d\vec{\xi}/dx_i)=0$, (i = 1; 2), equality (15) becomes

$$\frac{\partial\Phi[x_1,x_2]}{\partial x_i}=q_i+\frac{x_i-\xi_i}{|\vec{x}-\vec{\xi}|}, \quad \mathrm{i}=1;2.$$

Hence the following important sentence results.

Theorem: If the system of equations

$$l_1^*(\xi_1,\xi_2)+q_1=0; l_2^*(\xi_1,\xi_2)+q_2=0 \tag{16}$$

has some solution (ξ_1,ξ_2) and $(\xi_1,\xi_2,g(\xi_1,\xi_2))\in\overline{S}$, then the values (x_1,x_2), expressed through this solution via formulae (14), are the stationary coordinates of a phase function $\Phi[x_1,x_2]$.

Thus, the detection of SPP in the aperture and finding their coordinates reduce to the effective solving of system of (16).

Turn now to this problem. Fix the unit vectors of radiation $\vec{R}^0$ and scanning $\vec{q}$

$$\vec{R}^0=\begin{pmatrix}\cos\varphi\cdot\sin\vartheta\\ \sin\varphi\cdot\sin\vartheta\\ \cos\vartheta\end{pmatrix}; \quad \vec{q}=\begin{pmatrix}\cos\alpha\cdot\sin\psi\\ \sin\alpha\cdot\sin\psi\\ \cos\psi\end{pmatrix} \tag{17}$$

where $\alpha,\varphi\in[0;2\pi)$; $\vartheta\in(0;\pi/2]$; $\psi\in[0;\pi/2)$.

Using (8), (16), (17) and denoting $\cos\varphi = \mathrm{c}$; $\sin\varphi = \mathrm{s}$, system of (16) takes the form

$$2g_{\xi_1}\left[(g_{\xi_1}\cdot c+g_{\xi_2}\cdot s)\sin\vartheta-\cos\vartheta\right]/\left(1+g_{\xi_1}^2+g_{\xi_2}^2\right)=A_1 \quad (18)$$
$$2g_{\xi_2}\left[(g_{\xi_1}\cdot c+g_{\xi_2}\cdot s)\sin\vartheta-\cos\vartheta\right]/\left(1+g_{\xi_1}^2+g_{\xi_2}^2\right)=A_2 \quad (19)$$

where

$$\left.\begin{array}{l} A_1 = -\sin\psi\cdot\cos\alpha+\sin\vartheta\cdot\cos\varphi \\ A_2 = -\sin\psi\cdot\sin\alpha+\sin\vartheta\cdot\sin\varphi. \end{array}\right\}$$

It follows from (18), (19) that:

$$g_{\xi_1} : A_1 = g_{\xi_2} : A_2. \quad (20)$$

Denote the value of ratio (20) as $w = w(\xi_1, \xi_2)$

$$g_{\xi_1} = A_1 w; \quad g_{\xi_2} = A_2 w \quad (21)$$

after substitution in (18) or (19), derive

$$(A^2 - \bar{A}\ \sin\ \vartheta)w^2 + 2\ \cos\ \vartheta\cdot w + 1 = 0$$

where $A^2 = A_1^2 + A_2^2; \overline{A} = A_1 c + A_2 s$, and, finally, the equation for w

$$(\sin^2\psi - \sin^2\vartheta)w^2 + 2\ \cos\ \vartheta\cdot w + 1 = 0$$

whence two solutions can be found

$$w_1 = -\frac{1}{\cos\vartheta+\cos\psi}$$
$$w_2 = -\frac{1}{\cos\vartheta-\cos\psi}.$$

Substituting $w = w_1$ in formulae (21), we obtain the first solution of system of (18), (19)

$$\left.\begin{array}{l} g_{\xi_1}^I = -\frac{A_1}{\cos\vartheta+\cos\psi} = \frac{\sin\psi\cdot\cos\alpha-\sin\vartheta\cos\varphi}{\cos\vartheta+\cos\psi} \\ g_{\xi_2}^I = -\frac{A_2}{\cos\vartheta+\cos\psi} = \frac{\sin\psi\cdot\sin\alpha-\sin\vartheta\sin\varphi}{\cos\vartheta+\cos\psi}; \end{array}\right\}. \quad (22.1)$$

Analogously, the second solution of (18), (19) can be obtained using w_2

$$\left.\begin{array}{l} g_{\xi_1}^{II} = -\frac{A_1}{\cos\vartheta-\cos\psi} = \frac{\sin\psi\cdot\cos\alpha-\sin\vartheta\cos\varphi}{\cos\vartheta-\cos\psi} \\ g_{\xi_2}^{II} = -\frac{A_2}{\cos\vartheta-\cos\psi} = \frac{\sin\psi\cdot\sin\alpha-\sin\vartheta\sin\varphi}{\cos\vartheta-\cos\psi} \end{array}\right\}. \quad (22.2)$$

Therefore, (18), (19) has two solutions: (22.1) and (22.2). Physical meaning of this ambiguity is determined by conditions to obey the desired solution beside (18), (19). Demonstrate this fact for the case when azimuthal angles of vectors $\vec{q}$ and $\vec{R}^0$ coincide: $\alpha = \varphi$. As is easy to see

$$g_{\xi_1}^I = -c\cdot tg\frac{\vartheta-\psi}{2}; \quad g_{\xi_2}^I = -s\cdot tg\frac{\vartheta-\psi}{2} \quad (23.1)$$
$$g_{\xi_1}^{II} = c\cdot ctg\frac{\vartheta+\psi}{2}; \quad g_{\xi_2}^{II} = s\cdot ctg\frac{\vartheta+\psi}{2}. \quad (23.2)$$

Look now at the expression for the component l_3^* derived from (8), (17)

$$l_3^* = \frac{1}{1+g_{\xi_1}^2+g_{\xi_2}^2}\left[\left(1-g_{\xi_1}^2-g_{\xi_2}^2\right)\cos\vartheta - 2\left(g_{\xi_1}\cdot c+g_{\xi_2}\cdot s\right)\sin\vartheta\right]. \quad (24)$$

Substitution of expressions (23.1) and (23.2) in (24) provides the following results: $\left(\vec{l}_3^*\right)^I = \cos\psi > 0$; $\left(\vec{l}_3^*\right)^{II} = -\cos\psi$.

However, the second variant contradicts the problem geometry: reflected beam directed along $\vec{l}^*$ is to be directed down to the antenna aperture. Thus, we have only one physically meaningful solution: $(g_{\xi_1}^{II}, g_{\xi_2}^{II})$.

IV. Numerical Results

Consider circular aperture of radius $\alpha = 5\lambda_0$ (λ_0 is the wavelength in the free space) with cosine-law amplitude distribution (represented by $\vec{E}$, $\vec{H}$ in (4)) placed symmetrically to the axis of a parabolic radome, whose surface equation is

$$x_3 = -\frac{1}{2P_0}\left(x_1^2+x_2^2\right) + h.$$

Parameters of the radome are: $\mathrm{P}_0 = 1.4\lambda_0$, the depth of the radome $h = 5.56\lambda_0$, the dielectric permittivity $\varepsilon_1 = 7$, and the thickness is matched for the normal incidence.

The unit polarization vector of currents in the aperture is $\vec{p}_0 = (0,1,0)$ $(\vec{E}_t = \vec{p}_0\cos[\pi\sqrt{x_1^2+x_2^2}/(2a)]e^{(\vec{q}\cdot\vec{x})})$.

Using formula (4) we calculated radiation patterns in H-plane ($\vec{p} = (0,1,0)$, $\vec{R}_0 = (\sin\gamma, 0, \cos\gamma)$; Fig. 5) and compared them with the radiation patterns of the aperture enclosed in a semi-spherical radome, with radius $r = 5.56\lambda_0$. Scanning was carried by vector $\vec{q} = (\sin\alpha, 0, \cos\alpha)$. All radiation patterns were normalized to the maximum of the radiation pattern without a radome.

Note, that the radiation pattern of a circular aperture without electrical scanning [Fig. 5(a)] coincides with the corresponding pattern of [27], where it was computed by exponential approximation of Bessel function.

A significant increasing of the sidelobe level in the far zone is observed in comparison with the radiation pattern of an aperture without radome in H-plane. Far sidelobes grow by 15–20 dB. This phenomenon is caused by reflections from the small area (less than 1% of surface of the radome inner wall), associated with a stationary phase points. For example, contribution of such area to the total reflected field for $\gamma = 70^\circ$ without scanning is equal to 85 per cents. The main lobe decreases by 0.6 dB.

The radiation pattern for a semi-spherical radome is much more distorted compared to a parabolic radome for the case of a cosine-law distribution; however, this effect is less explicit for a constant amplitude and phase distribution [Fig. 5(b)].

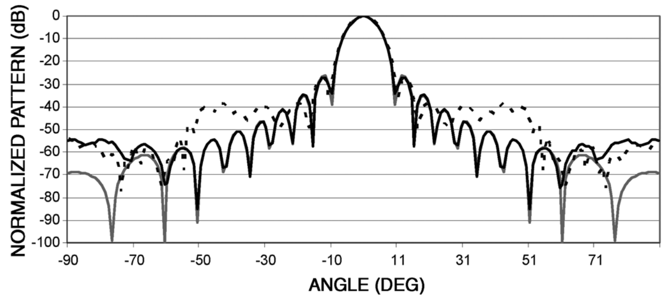
With increasing of scanning angle, the pattern symmetry is breaking and far sidelobes are broaden and considerably distorted [Fig. 5(c)].

V. Conclusions

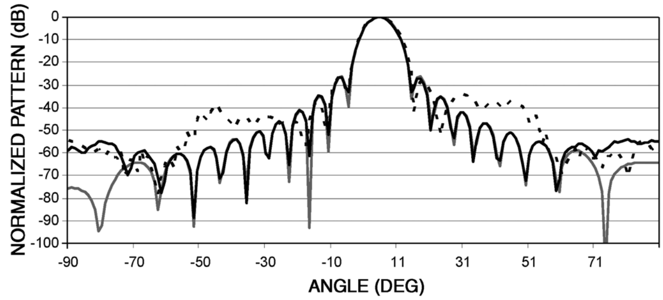
The offered method allows to calculate the radiation field of an aperture antenna in the presence of arbitrary radome or (with some modifications) of any scatterers. It is equally applicable to study fields both in the near and far zones of a radome.

For correct calculation of GO (GTD) passage of a wave through the radome, the caustic behavior is analyzed.

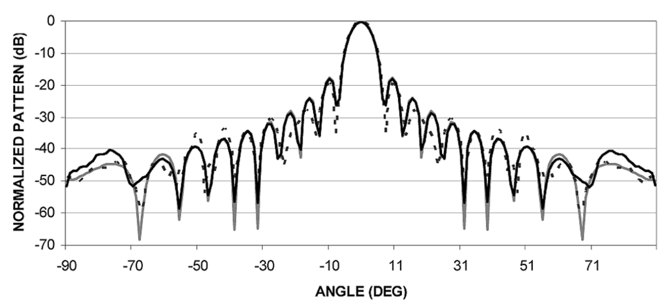
The existence and coordinates of stationary phase points of the reflected field in the aperture Σ_0 can be recognized via the following developed algorithm.



(a)



(b)



(c)

Fig. 5. Radiation patterns of the circular aperture antenna with the parabolic radome (black line), the semi-spherical radome (black dashed line), and without radome (gray line) in H-plane. (a) Without scanning for cosine amplitude distribution. (b) Scanning under an angle of 5° for cosine amplitude distribution. (c) Without scanning for constant distribution.

1) With given vectors $\vec{R}^0$ of radiation and $\vec{q}$ of scanning, solve the system of (18), (19) and choose the solutions (points $\vec{\xi}(\xi_1, \xi_2, g)$) lying in the domain $\overline{S} \subset S$.
2) Using formulae (19), determine the points of plane $\mathrm{x}_3 = 0$ with coordinates $(x_1, x_2, 0)$ corresponding to each found solution $\vec{\xi} \in \overline{S}$ of (18), (19).
3) Among these points choose those that belong to the domain Σ_0 (i.e. the aperture of the considered antenna system) and thus obtain the desired set of stationary phase points of the field reflected from S.

A practical significance of these derivations is to build a grid of stationary phase points (SPP) whose local neighborhoods contribute significantly to the lateral radiation of the antenna system. In doing so may open opportunity of compensation of this contribution by technical devices. Neighborhoods of SPPs contribute significantly to the level of the reflected field (60–85% of the total value of reflections). Sizes of these areas are quite small and do not overcome 2% of an aperture size. These results can be also used for field evaluation by the method of stationary phase.

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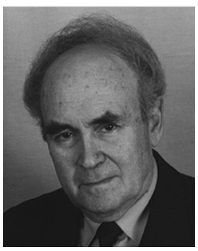

Ilya Vladimirovich Sukharevsky received the M.Sc. degree in mechanics and the Ph.D. degree in mathematical physics from Kharkov State University, Ukraine, and the D.Sc. degree in mathematical physics from the Military Academy of Air Defence, Kharkiv, in 1950, 1954, and 1967, respectively.

He served in the Soviet Army during the Second World War. He was with Kharkov Polytechnical University from 1950 to 1958 and with Military Academy of Air Defence, Kharkiv, Ukraine, from 1959 to 1993. He was a Professor at Kharkiv Military University, Kharkiv, from 1993 to 2000. For 33 years from 1960 to 1993, he was the head of the Mathematics School of Military Academy of Air Defence. He held lectures on various mathematical courses (integral equations, mathematical statistics, computational mathematics, etc.) at the Mathematics and Mechanics Department of Kharkov State University and, for more than a decade, he was the head of the State Certification Commission. His early research interests (1953–1958) were mostly concentrated on the bounded problems and integral equation of the potential theory with applications to the hydrodynamics and elasticity theory. After 1958 he, in cooperation with Prof. A. Ya. Povzner, held a cycle of fundamental research of wave fronts and discontinuities of Green's function on them in the case of the boundary value problem in a domain wholly illuminated by a source. This research became one of the first basic results in the microlocal analyses of partial differential equations, and have found wide use in acoustics and electrodynamics, in the problems of echolocation and radars. The vast number of his publications were dedicated to diffraction problems on radar-transparent stratified structures. The range of important asymptotic expansions with applications to antenna systems and radomes were obtained. The other direction of his scientific research was connected with operator, integral, in particular, equations. He was the author of papers dedicated to the stability of solutions of operator equations in Banach space, the stability of solutions of Fredholm equations under the discontinuous kernel variation, and the second kind integral equations of diffraction on open conducting surfaces.

Prof. Sukharevsky was a member of the Methodological Commission of the Defense Ministry of the USSR and is a the permanent member of Presidium of Kharkov Mathematical Society since 1972. He was awarded the title of the Honoured Worker of Education of Ukraine in 1991.

Sergey E. Vazhinsky was born in 1961. He received the B.Sc. degree in electrical engineering from Kyiv Engineering Missile Academy, Ukraine, in 1985, and the M.Sc. and the Ph.D. degrees from the Military Academy of Air Defence and Kharkov Military University, in 1995 and 1997, respectively.

He is now an Assistant Professor in the National University of Defence, Kiev, Ukraine. His current research interests include electronic systems and antennas.

Ilya O. Sukharevsky (S'08) was born in Kharkov, Ukraine, in 1984. He received the B.Sc. and the M.Sc. degrees in mathematical physics from Kharkov National University, Ukraine, in 2005 and 2006, respectively, where he is currently working toward the Ph.D. degree.

He is now with the Institute for Radiophysics and Electronics of the National Academy of Sciences of Ukraine, Kharkov, Ukraine, as a Junior Scientist. His scientific interests are in the areas of antennas and computational electromagnetics.