

*The object of research is the fire safety subsystem of hydrogen storage and supply systems. The subject of the study is the efficiency index of the fire safety subsystem of hydrogen storage and supply systems for different modes of its operation. As such an efficiency indicator, the conditional probability that the fire safety subsystem correctly recognizes the actual state of the hydrogen storage and supply system is used. The fire safety subsystem functions under the control mode and under the test mode. Mathematical models of the operation of the fire safety subsystem were built for such modes, based on the use of graph theory. The weight matrices of these graphs include the completeness of control or testing and the intensity of transition of the fire safety subsystem from one state to another. Determination of the effectiveness of such a subsystem – reliability of functioning – is carried out using Kolmogorov's equations. It is shown that during the testing of hydrogen storage and supply system, the probability of its being in a fire-safe state has a maximum. It is shown that with values of completeness of control (testing) that do not differ from 1.0, the effectiveness of the functioning of the fire safety subsystem is invariant with respect to the mode of its functioning. With values of completeness of control (testing), which are significantly different from 1.0, the functioning of the fire safety subsystem under the testing mode is more effective.*

*The identified features of the functioning of the fire safety subsystem make it possible in practice to implement an optimal or adaptive algorithm for the functioning of such subsystems. For example, with the appropriate selection of testing parameters, the fire safety subsystem provides determination of the location of the hydrogen storage and supply system with maximum probability*

*Keywords: hydrogen systems, evaluation efficiency, fire safety, Kolmogorov equation, graph theory*

# DETERMINING THE FUNCTIONING EFFICIENCY OF A FIRE SAFETY SUBSYSTEM WHEN OPERATING THE HYDROGEN STORAGE AND SUPPLY SYSTEM

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## 1. Introduction

Hydrogen is becoming an increasingly viable environmentally friendly source of energy [1]. It is expected to become one of the most important energy carriers in the 21<sup>st</sup> century. One of the main keys to this is the safe, compact, and cost-effective hydrogen storage provided by Hydrogen Storage Systems (HSS). At the same time, solid-state hydrogen storage and supply systems are becoming an increasingly attractive option for hydrogen applications [2]. However, during their operation, the influence of various factors, such as material damage, design defects, personnel errors or external collisions and shocks, can lead to the emergence of fire and explosive situations [3]. Alleviation of these factors is possible with the help of the fire safety subsystem.

Therefore, studies aimed at increasing the efficiency of the operation system of hydrogen storage and supply systems, which include their fire safety subsystems, are relevant.

## 2. Literature review and problem statement

In [4] it is noted that there are serious safety problems associated with the hydrogen process. In this context, the hydrogen explosion at the Fukushima nuclear power plant (Japan) in 2011, incidents at a hydrogen filling station in Norway (June 2019), and a chemical plant in California (USA, 2020) should be noted. These incidents resulted in fires and explosions that caused serious material losses. The safety of hydrogen systems is considered in two aspects: safety

related to the properties of hydrogen and safety related to the operation of hydrogen systems. The specific properties of hydrogen [5] include low molar mass, which indicates easy penetration and diffusion. The lack of smell and color leads to difficulties in its detection. A minimum ignition energy of 0.02 mJ indicates that hydrogen is easily ignited. The wide range of flammability (volume fraction of hydrogen (4.1-74.1) %) pre-determines the need to define the value of this parameter for specific conditions of its use. One of the effective methods of such determination can be obtaining predictive estimates of the flammability of hydrogen. In [6], three directions are used to predict the flammability of hydrogen: experimental, numerical, and analytical. Experimental methods, as a rule, are carried out under laboratory conditions and with their help estimates of limit concentrations are obtained. Numerical methods require substantiation of the accuracy of determining forecast estimates. In addition, these methods, as well as analytical methods, require answers to questions regarding the adequacy of the description of hydrogen processes. In [7] it is noted that effective risk assessment is crucial in preventing fires and explosions in hydrogen systems. Using the example of a refueling station using hydrogen, the Accident Risk Assessment for Industrial Systems (ARAMIS), and Computational Fluid Dynamics (CFD) methods were used to derive risk levels at the station and on the road near it. The magnitude of these risks was  $5.8 \cdot 10^{-5}$  and  $3.37 \cdot 10^{-4}$ , respectively, which is acceptable according to ALARP recommendations. This approach to obtaining a quantitative risk assessment (QRA) is integral and does not ensure the identification of the most dangerous factors. In [8], the results of the analysis of risk factors that lead to hydrogen logistics incidents are given. Network analysis was used to identify significant factors. A mathematical model was built that belongs to the class of regression models and allows obtaining quantitative estimates of the impact of each of the factors on a hydrogen logistics incident. But this approach does not make it possible to get an answer to the question about the possibility of the appearance of one or another factor for the realization of a hydrogen logistics incident. In [9], using the Bayesian model (BN), the risk of fire and explosion, as well as the degree of loss of personnel of the on-board high-pressure hydrogen system, are determined. When hydrogen is released, the probability of an explosion is  $6.79 \cdot 10^{-5}$ , the probability of a jet fire is  $1.53 \cdot 10^{-4}$ , and the probability of a fireball is  $5.38 \cdot 10^{-8}$ . It is shown that with extensive damage there is a probability of explosion that can reach 0.87. It should be noted that the derivation of QRA depends significantly on the accuracy of numerical models, such as hydrogen leakage, diffusion, combustion, and explosion, as well as on the setting of simulation conditions, such as mesh size and boundary conditions. As a result, the question regarding the effectiveness of the QRA method needs to be answered. In the vast majority of cases, QRA is obtained theoretically using numerical modeling methods. In [10], the results of research conducted within the framework of MOZEES (Norwegian research center for environmentally friendly technologies and zero-emission transport) are reported. The purpose of the research was to assess whether the risk associated with hydrogen systems is equivalent to the risk for conventionally fueled vessels. It has been proven that the risk does not exceed (0.5÷1.0) fatalities per 109 passenger-kilometers. For this, a technique was used that is focused on traditional fuels, as a result of which a question arises regarding the reliability of obtaining results. In [11], the oc-

currence of fire-explosive situations during the operation of tanks as elements of hydrogen storage and supply systems was determined. The results of these studies provide an answer to the question of «what if», but do not answer the question of what is the probability of conditions that lead to tank explosions. In [12], the fire hazard of hydrogen fuel cell vehicles (HFCVs) was analyzed using a method combining failure mode and effect analysis (FMEA) with a risk assessment matrix. This approach provides a more intuitive analysis and allows the identification of potential failure modes in the system so that mitigation measures can be taken in advance. It should be noted that the FMEA analysis is performed without taking into account the influence of people, the environment and the interaction between different subsystems or components. Qualitative analysis of FMEA results is used to assess the severity of the risk, the reliability of the obtained results is not assessed. Quantitative risk assessments – QRA of the latest model hydrogen filling station (HRS) were obtained in [13]. Advances in the technology of the components or facilities used in HRS, accumulated experience in their operation, as well as new knowledge about the safe use of hydrogen in HRS prompted a QRA to update these results. It should be noted that when assessing quantitative risks, it is necessary to assess the frequency of these risks. Risk frequency analysis for HRS has a significant problem because there is not a sufficient body of statistical data on failure rates. This was due to the fact that HRS has only been used for a short time. To overcome this situation, the frequencies were estimated using data from chemical plants. The reliability of the obtained results was not evaluated. In [14], the methodology for quantitative assessment of the risk of rupture of a hydrogen tank is presented. The analysis of the consequences is carried out for the rupture of a tank with a capacity of 62.4 liters, which is under a pressure of 70 MPa during a fire. Two tank state-of-charge (SoC) scenarios are considered – SoC=99 % and SoC=59 %. The risk assessment uses the number of fatalities per vehicle per year, the damage per accident, and the fire resistance rating (FRR) of the hydrogen tank. Such an approach can be used when forming requirements for the response time of fire departments to a fire. The methodology of this approach to risk assessment does not provide an answer to the question of the possibility of the occurrence (creation) of a fire-explosive situation and obtaining an assessment of such a possibility. A complete risk assessment procedure consists of hazard identification, consequence modeling, quantification of frequency data, and risk characteristics. A useful technique for identifying hazard scenarios is qualitative risk assessment. In [15], an on-board hydrogen storage and supply system was studied using HAZOP and FMEA methods. The generated scenarios were obtained based on four processes: hydrogen filling, hydrogen storage, hydrogen supply, and hydrogen pumping. Risk assessment was carried out with and without security measures by creating a risk matrix. The degree of confidence in the obtained results is not given. The lack of actual data on explosions to confirm the adequacy of theoretical prediction models and numerical simulations can be compensated by experimental data. In [12], the data on the catastrophic consequences of rupture of hydrogen storage tanks during a fire are given. As a rule, hydrogen is stored in cylinders under a pressure of 35 MPa or 70 MPa. In particular, Toyota Mirai is equipped with two tanks with a pressure of 70 MPa ( $V_1=60$  l and  $V_2=62.4$  l). The results of the experiments showed that the danger of the explosion of hydrogen

storage cylinders is related to the combined contribution of the physical and chemical energy of the explosion. After the explosion of the balloon, the duration of the hydrogen-air deflagration was 2.0 s, and the maximum diameter of the fireball was 4.48 m. The maximum distance of the fragments was 46.0 m. It should be noted that studies of this type are of an exclusive nature, and their results may be used to form trends to ensure safe operation of vehicles on an intuitive level. The same type of research includes [16], in which the results regarding explosions in the hydrogen storage and supply system are given. Data are provided on the conditions under which explosions may occur in such systems, but no data are provided on the probability of their occurrence. One of the ways out of this situation is to establish a connection between fire and explosion hazard indicators and reliability indicators. In [17], a set of mathematical models was built to determine the probability of the appearance of a combustible medium in the hydrogen storage and supply system. This approach is based on the use of probabilities of failure-free operation of the main elements of the hydrogen storage and supply system. The need to take into account parametric failures of the main elements of the hydrogen storage and supply system is substantiated. Failure to take into account the parametric failures of the main elements of the system results in an error of up to 30 %. The peculiarity of this approach is that there are practically no data on the reliability of the hydrogen storage and supply system. This especially applies to solid-state hydrogen storage and supply systems [18]. One of the ways to get out of this situation is to use non-traditional methods to obtain estimates of reliability indicators of storage and supply systems. An example of such an approach is given in [19], in which its amplitude-frequency characteristic is used to determine the probability of failure-free operation of the main element of the hydrogen storage and supply system – the gas generator. This characteristic is determined numerically. It is shown that the probability of gas generator failure is  $2 \cdot 10^{-4}$ . It should be noted that this approach to obtaining the reliability indicator is multi-stage, as a result of which errors occur at each of the stages during its implementation, the values of which are integrated. In [20], the amplitude-frequency and phase-frequency characteristics of the gas generator are used to obtain reliability indicators of the gas generator of the hydrogen storage and supply system. It is shown that, with a probability of 0.9973 and 0.9812, respectively, its amplitude-frequency and phase-frequency characteristics will not differ from the nominal values by more than 5.0 % at the time of starting the gas generator. The latter limits the capabilities of this method. Such restrictions are removed in [21], in which the dependence of the pressure in its cavity on the working time at a random time is used to determine the reliability index of the gas generator (and the level of its fire hazard). The probability of trouble-free operation of the gas generator of the hydrogen storage and supply system is determined using the Laplace function, one of the parameters of which is the rate of change of pressure in the gas generator cavity. The value of this parameter depends on several factors, in particular, on the temperature in the cavity of the gas generator, which is not controlled. Paper [22] presents the methods for determining the parameters of the gas generator of the hydrogen storage and supply system, which are used to determine the assessment of its reliability indicators. The peculiarity of these methods is that they are based on the use of the transient function of the gas generator, which can be determined during its operation. It should be

noted that all methods for determining reliability indicators given in [19–22] are not applicable to the entire hydrogen storage and supply system, but only to its gas generator. This approach requires an answer to questions about its correctness. In [23], a system for monitoring the fire-hazardous state of the hydrogen storage and supply system is used to answer this question. It is shown that with the completeness of control, the value of which is 0.5, the probability of trouble-free operation of such a control system should be 0.995, while the probability of a fire-safe state of the hydrogen storage and supply system is 0.9. It should be noted that control systems of this type belong to passive control systems, that is, the results of control of the level of fire danger of hydrogen storage and supply systems are not used to improve their fire safety properties. The use of active systems for monitoring the fire safety condition of hydrogen storage and supply systems (fire safety subsystems) opens up new opportunities to ensure the necessary level of their fire safety. In such subsystems of fire safety of the hydrogen storage and supply system, based on the control results, restoration of the fire-safe state of the system is envisaged after the detection of the presence of this system in a fire-hazardous state.

It should be noted that a characteristic feature of the process of determining the level of fire danger of hydrogen storage and supply systems is the absence of an option aimed at determining the degree of confidence in the obtained results. This degree of confidence characterizes the effectiveness of the methods and tools used to obtain estimates of the level of fire danger of hydrogen storage and supply systems.

All this gives reason to assert that it is expedient to carry out research aimed at determining the efficiency of the functioning of fire safety subsystems of hydrogen storage and supply, during the operation of which the option of restoring their fire safety condition is implemented.

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### 3. The aim and objectives of the study

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The purpose of our study is to substantiate the effectiveness of the fire safety subsystem during the operation of solid-state hydrogen storage and supply systems under control and testing modes, provided that such systems are restored. In practice, this opens up opportunities for the implementation of optimal algorithms for the functioning of fire safety subsystems of hydrogen storage and supply systems.

To achieve this goal, the following tasks must be solved:

- to determine the indicator that characterizes the degree of confidence in the results obtained with the help of the fire safety subsystem of the hydrogen storage and supply system during its operation;
- to devise a mathematical description for the operation of the fire safety subsystem in the state control mode of hydrogen storage and supply systems and to determine the reliability of the result of this control;
- to build a mathematical model to describe the functioning of the fire safety subsystem of the hydrogen storage and supply system under their testing mode, to determine the reliability of the test result and to compare it with the control results of such systems.

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### 4. The study materials and methods

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The object of research is the fire safety subsystem of hydrogen storage and supply systems. The subject of the study

is the efficiency index of the fire safety subsystem of hydrogen storage and supply systems for different modes of its operation. The fire safety subsystem is the implementation of a set of organizational and technical measures aimed at ensuring the necessary level of fire safety of hydrogen storage and supply systems during their operation. The research material is hydrogen in its solid-state storage and supply system. An example of such systems are the systems designed by the Institute for Engineering Problems at the National Academy of Sciences of Ukraine (IPMash NAS, Ukraine, Kharkiv) based on hydro-reactive compositions. The main research hypothesis assumes that hydrogen storage and supply systems belong to the class of renewable systems.

The main assumptions are:

- the stationary mode of functioning of the fire safety subsystem of hydrogen storage and supply systems is considered;
- to determine the performance indicator of the fire safety subsystem, an indicator in terms of probability theory is used.

Graph theory together with matrix theory is used to describe the location of the fire safety subsystem in various states of its functioning. Kolmogorov's equations are used to build a mathematical model that formalizes the effectiveness of the fire safety subsystem. These equations are given in matrix form.

### 5. Results of research into the effectiveness of the functioning of the fire safety subsystem of the hydrogen storage and supply system

#### 5.1. Determining the degree of confidence in the results of fire safety subsystem performance

The operating system of hydrogen storage and supply systems includes fire safety subsystems. With the help of such a subsystem, a number of tasks are solved, which, in particular, include:

- control of the state of fire danger of hydrogen storage and supply systems;
- testing of hydrogen storage and supply systems;
- restoring the fire-safe state of hydrogen storage and supply systems.

The hydrogen storage and supply system can be in two states: fire-safe – B, or fire-hazardous – NN. As a result of control or testing by the fire safety subsystem, the hydrogen storage and supply system is recognized as:

- fireproof (with probability  $P_{BB}(t)$ );
- fire-hazardous (with probability  $P_{NN}(t)$ );
- fire-safe in the actual state that corresponds to the fire-hazardous state (with probability  $P_{NB}(t)$  – the customer's risk);
- fire-hazardous in the actual state that corresponds to the fire-safe state (with probability  $P_{BN}(t)$  – the supplier's risk).

All these events form a complete group of events, as a result of which the following occurs:

$$P_{BB}(t) + P_{NN}(t) + P_{NB}(t) + P_{BN}(t) = 1.0. \tag{1}$$

The effectiveness of the fire safety subsystem can be assessed by the degree of confidence in the results obtained with its help, that is, by the reliability of its functioning. The reliability of the functioning of the fire safety subsystem is the conditional probability  $D(t)$  that this subsystem correctly recognizes the actual state of the hydrogen storage and supply system. This conditional probability is defined by the expression:

$$D(t) = \left[ P_{BB}(t) + P_{NN}(t) \right] \left[ P_{BB}(t) + P_{NN}(t) + P_{NB}(t) + P_{BN}(t) \right]^{-1}, \tag{2}$$

which, subject to (1), takes the form of:

$$D(t) = P_{BB}(t) + P_{NN}(t). \tag{3}$$

The indicator  $D(t)$  refers to the general efficiency indicators of fire safety subsystems of hydrogen storage and supply systems. In addition to the general indicators of the efficiency of such subsystems, partial indicators can be used that characterize the completeness of control  $\omega_c$  or the completeness of testing  $\omega_r$  of the state of the hydrogen storage and supply system. The values of these indicators lie in the range of  $0 \neq 1.0$ . The control mode differs from the testing mode in that, in the first case, the parameters of the hydrogen storage and supply system are determined, which characterize the level of its fire hazard (or fire safety) directly under the mode of its regular operation. In the second case, a test effect is carried out on the hydrogen storage and supply system, based on the reaction to which the fire safety status of this hydrogen storage and supply system is determined.

#### 5.2. Mathematical description of the operation of the fire safety subsystem under control mode

Fig. 1 shows the formalization of the functioning algorithm of the fire safety subsystem under the control mode of the hydrogen storage and supply system.

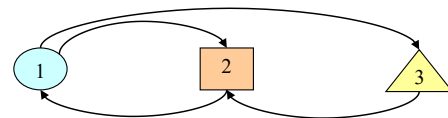


Fig. 1. Graph of states of fire safety subsystem (control mode): 1 – corresponds to the BB state; 2 – corresponds to the NN state; 3 – corresponds to the NB state

This formalization represents a graph of states in the fire safety subsystem. Such a graph of states corresponds to the weight matrix, which has the form:

$$G_s = \begin{pmatrix} 0 & \mu_s & 0 \\ \omega_c \lambda_s & 0 & \omega_c \lambda_s \\ (1 - \omega_c) \lambda_s & 0 & 0 \end{pmatrix}. \tag{4}$$

In this weight matrix,  $\lambda_s$  is the transition intensity of the hydrogen storage and supply system from the BB state to the NN state, and  $\mu_s$  is the restoration of the BB state after the hydrogen storage and supply system was recognized in the NN state. For  $\mu_s$  the following holds:

$$\mu_s = \tau_s^{-1}, \tag{5}$$

where  $\tau_s$  is the system recovery time.

In this case, the efficiency of the fire safety subsystem will be evaluated by an indicator, the expression for which in the stationary mode has the form:

$$D = \sum_{i=1}^2 P_i, \tag{6}$$

where  $P_1, P_2$  are the probabilities of finding the fire safety subsystem in states 1 and 2, respectively. The probabilities  $P_i$  ( $i=1..3$ ) are determined by the roots of the Kolmogorov equation:

$$C \cdot P = A, \tag{7}$$



where:

$$\mathbf{P} = (P_1 \ P_2 \ P_3)^T; \quad (8)$$

$$\mathbf{A} = (0 \ 0 \ 0 \ 1)^T; \quad (9)$$

$$\mathbf{C} = \begin{pmatrix} -\lambda_s & \mu_s & 0 \\ \omega_c \lambda_s & -\mu_s & \omega_c \lambda_s \\ (1-\omega_c)\lambda_s & 0 & -\omega_c \lambda_s \\ 1 & 1 & 1 \end{pmatrix}. \quad (10)$$

Under conditions (9) and (10), the following holds for the probabilities  $P_1, P_2$ :

$$P_1 = \left( \frac{1}{\omega_c} + \frac{\lambda_s}{\mu_s} \right)^{-1}; \quad P_2 = \frac{\lambda_s}{\mu_s} \left( \frac{1}{\omega_c} + \frac{\lambda_s}{\mu_s} \right)^{-1}, \quad (11)$$

as a result, the reliability of the functioning of the fire safety subsystem is described by the expression:

$$D = \sum_{i=1}^2 P_i = \omega_c \left( 1 + \frac{\lambda_s}{\mu_s} \right) \left( 1 + \omega_c \frac{\lambda_s}{\mu_s} \right)^{-1}. \quad (12)$$

Fig. 2 shows the dependence of reliability of D on the completeness of control  $\omega_c$  and on the relationship  $\lambda_s \mu_s^{-1}$ , which characterizes the level of fire danger of the hydrogen storage and supply system and the level of perfection of means of restoring its condition.

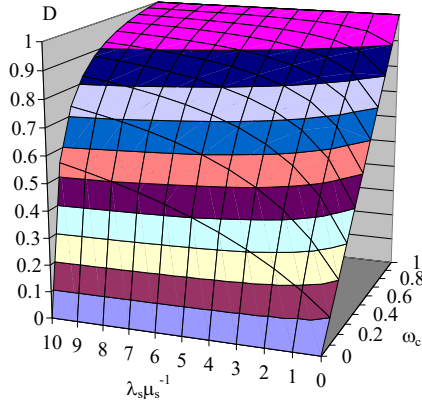


Fig. 2. Dependence of the reliability of the operation of the fire safety subsystem under the control mode on the completeness of control  $\omega_c$  and the ratio of intensities  $\lambda_s$  and  $\mu_s$

It should be noted that increasing the completeness of the control over the hydrogen storage and supply system or reducing the time to restore its condition ensures an increase in the efficiency of the fire safety subsystem. In particular, the following applies:

$$\lim_{\omega_c \rightarrow 1.0} D = 1.0; \quad \lim_{\tau_s \rightarrow 0} D = \omega_c, \quad (13)$$

as a result, it is advisable to increase the efficiency of the fire safety subsystem in the first place by increasing the completeness of control over the hydrogen storage and supply system.

### 5. 3. Mathematical notation of the functioning of the fire safety subsystem under the testing mode

Fig. 3 shows the state graph of the fire safety subsystem under the testing mode of the hydrogen storage and supply system.

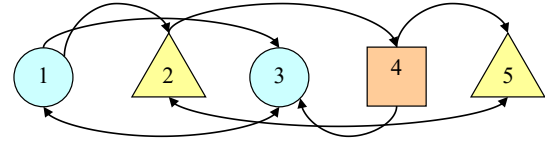


Fig. 3. Graph of states of the fire safety subsystem (testing mode): 1, 3 – corresponds to the state of BB; 2, 5 – corresponds to the NB state; 4 – corresponds to the NN state

The weight matrix for this graph is:

$$\mathbf{G}_T = \begin{pmatrix} 0 & 0 & \mu_T & 0 & 0 \\ \lambda_s & 0 & 0 & 0 & \mu_T \\ \lambda_T & 0 & 0 & \omega_c \mu_s & 0 \\ 0 & \lambda_T & 0 & 0 & 0 \\ 0 & 0 & 0 & (1-\omega_c) \mu_s & 0 \end{pmatrix}, \quad (14)$$

where  $\lambda_T$  is the intensity of the testing period;  $\mu_T$  is the testing intensity.

For these parameters, the following applies:

$$\lambda_T = T_T^{-1}; \quad \mu_T = \tau_T^{-1}, \quad (15)$$

where  $T_T, \tau_T$  are the period and time of testing the hydrogen storage and supply system, respectively.

The effectiveness of the functioning of the fire safety subsystem in this case is determined by the expression:

$$D = P_1 + P_3 + P_4, \quad (16)$$

where  $P_i$  ( $i=1, 3, 4$ ) are the roots of Kolmogorov equations (7). The components of this equation are determined by the expressions:

$$\mathbf{P} = (P_1 \ P_2 \ P_3 \ P_4 \ P_5)^T; \quad (17)$$

$$\mathbf{A} = (0 \ 0 \ 0 \ 0 \ 0 \ 1)^T; \quad (18)$$

$$\mathbf{C} = \begin{pmatrix} -(\lambda_s + \lambda_T) & 0 & \mu_T & 0 & 0 \\ \lambda_s & -\lambda_T & 0 & 0 & \mu_T \\ \lambda_T & 0 & -\mu_T & \omega_c \mu_s & 0 \\ 0 & \lambda_T & 0 & \mu_s & 0 \\ 0 & 0 & 0 & (1-\omega_c) \mu_s & -\mu_T \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}. \quad (19)$$

The roots  $P_i$  ( $i=1, 3, 4$ ) of the Kolmogorov equations with components (17) to (19) are determined by the expressions:

$$P_1 = M^{-1}(\lambda_s, \mu_s, \lambda_T, \mu_T, \omega_c); \quad (20)$$

$$P_3 = \frac{\lambda_s + \lambda_T}{\mu_T} M^{-1}(\lambda_s, \mu_s, \lambda_T, \mu_T, \omega_c); \quad (21)$$

$$P_4 = \omega_T^{-1} \frac{\lambda_s}{\mu_s} M^{-1}(\lambda_s, \mu_s, \lambda_T, \mu_T, \omega_c), \quad (22)$$

where:

$$M(\lambda_s, \mu_s, \lambda_T, \mu_T, \omega_c) = 1 + \frac{\lambda_T}{\mu_T} + \omega_c^{-1} \left( \frac{\lambda_s}{\mu_s} + \frac{\lambda_s}{\mu_T} + \frac{\lambda_s}{\lambda_T} \right). \quad (23)$$

After combining (16) and (20) to (23), the expression for the reliability of the functioning of the fire safety subsystem under the test mode takes the form:

$$D = \left( 1 + \frac{\lambda_s + \lambda_T}{\mu_T} + \omega_T^{-1} \frac{\lambda_s}{\mu_s} \right) \times \left[ 1 + \frac{\lambda_T}{\mu_T} + \omega_T^{-1} \left( \frac{\lambda_s}{\mu_s} + \frac{\lambda_s}{\mu_T} + \frac{\lambda_s}{\lambda_T} \right) \right]^{-1} \tag{24}$$

It should be noted that unlike the operation of the fire safety subsystem under the control mode, the reliability of its operation under the test mode even at  $\omega_T=1.0$  does not reach the level of  $D=1.0$ . This follows from (24) and is due to the presence of an additive component  $\lambda_s \lambda_T^{-1}$  in the denominator of the expression for reliability  $D$ . The absolute maximum for the efficiency of the fire safety subsystem can be achieved under the following condition:

$$\omega_T \rightarrow 1.0; \lambda_s \lambda_T^{-1} \rightarrow 0. \tag{25}$$

The second condition is equivalent to increasing the testing frequency of the hydrogen storage and supply system using the fire safety subsystem. If in this case the testing time can be reduced, then the testing of the hydrogen storage and supply system can be carried out with the completeness of the test, the value of which is different from unity.

The conditions under which the hydrogen storage and supply system will correspond to the BB state with the maximum probability value, i.e., at  $P_1=P_{1max}$ , were determined.

If expressions (15), (20), and (23) are combined, then the  $P_{1max}$  value will correspond to the condition:

$$\frac{\partial P_1}{\partial T_T} = 0, \tag{26}$$

from which the ratio follows:

$$T_T = (\omega_T \tau_T \lambda_s^{-1})^{0.5}. \tag{27}$$

The ratio between the test period  $T_T$  and the test time  $\tau_T$  in the form of (27) is the condition under which the probability  $P_1$  reaches a maximum.

The effectiveness of the functioning of the fire safety subsystem in this case according to (24) is determined as follows:

$$D_m = \left[ 1 + \frac{\lambda_s}{\mu_T} + \left( \omega_T^{-1} \frac{\lambda_s}{\mu_T} \right)^{0.5} + \omega_T^{-1} \frac{\lambda_s}{\mu_s} \right] \times \left[ \left[ 1 + \left( \omega_T^{-1} \frac{\lambda_s}{\mu_T} \right)^{0.5} \right]^2 + \omega_T^{-1} \frac{\lambda_s}{\mu_T} \right]^{-1}. \tag{28}$$

The ratio  $\lambda_s \mu_T^{-1}$  characterizes the fire hazard level of the hydrogen storage and supply system and the level of perfection of its testing means. At a high level of perfection of the fire safety subsystem, which is equivalent to  $\tau_T \rightarrow 0$  or  $\mu_T \rightarrow \infty$ , the following occurs:

$$\lim_{\tau_T \rightarrow 0} D_m = 1.0. \tag{29}$$

Such a fire safety subsystem during testing of the hydrogen storage and supply system ensures determination of its fire safety status with absolute reliability. The probability of finding the hydrogen storage and supply system in this state

is the maximum possible, the value of which is determined by the expression:

$$P_{1max} = \left( 1 + \omega_T^{-1} \frac{\lambda_s}{\mu_s} \right)^{-1}. \tag{30}$$

It should be noted that for fire safety subsystems with a long testing time, the following takes place:

$$D_m \rightarrow \omega_T. \tag{31}$$

The nature of change in the reliability of the functioning  $D_m$  of the fire safety subsystem depending on the ratios  $\lambda_s \mu_T^{-1}$  and  $\lambda_s \mu_s^{-1}$  is shown in Fig. 4 at  $\omega_T=0.3$  and in Fig. 5 at  $\omega_T=0.9$ .

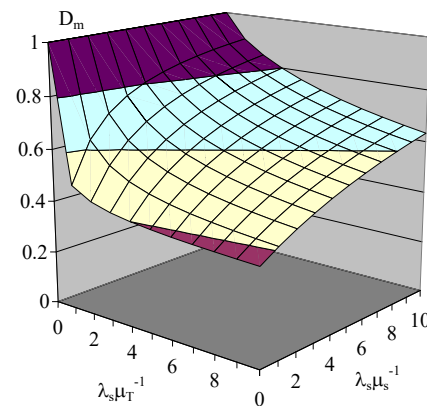


Fig. 4. Dependence of the reliability of the fire safety subsystem operation on parameters  $\lambda_s \mu_T^{-1}$  and  $\lambda_s \mu_s^{-1}$  with complete testing  $\omega_T=0.3$

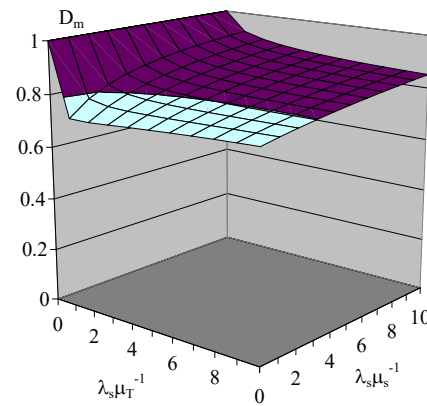


Fig. 5. Dependence of the reliability of the fire safety subsystem operation on parameters  $\lambda_s \mu_T^{-1}$  and  $\lambda_s \mu_s^{-1}$  with complete testing  $\omega_T=0.9$

A comparison of these dependences shows that at small values of the parameter  $\omega_T$  there is a greater sensitivity of the reliability of the functioning of the fire safety subsystem  $D_m$  relative to the change in parameters  $\lambda_s \mu_T^{-1}$  and  $\lambda_s \mu_s^{-1}$ . At the same time, it should be noted that the sensitivity of the reliability  $D_m$  is more significant relative to the change of the parameter that characterizes the level of perfection of the means of testing the storage system and hydrogen supply.

To compare the effectiveness of the functioning of the fire safety subsystem under the modes of control and testing of hydrogen storage and supply systems, the data given in Tables 1, 2 are used. Table 1 is constructed according to expression (12), and Table 2 – according to expression (28).

Table 1

Value of reliability of the functioning of the fire safety subsystem under control mode

$\omega_c$	$\lambda_s \mu_s^{-1}$			
	0.01	0.1	1.0	10.0
0.3	0.30	0.32	0.46	0.83
0.9	0.90	0.91	0.95	0.99

Table 2

Value of reliability of the functioning of the fire safety subsystem under test mode

$\omega_T$	$\lambda_s \mu_T^{-1}$	$\lambda_s \mu_s^{-1}$			
		0.01	0.1	1.0	10.0
0.3	0.01	0.86	0.88	0.96	0.99
	0.1	0.66	0.71	0.86	0.98
	1.0	0.48	0.50	0.63	0.90
	10.0	0.36	0.37	0.41	0.63
0.9	0.01	0.91	0.92	0.95	0.99
	0.1	0.81	0.82	0.88	0.97
	1.0	0.72	0.73	0.78	0.92
	10.0	0.75	0.76	0.77	0.85

Analysis of the data given in these tables reveals that with values of completeness of control (testing) that are significantly different from unity:

- the functioning of the fire safety subsystem is more effective under the mode of testing the state of the hydrogen storage and supply system;

- the increase in the efficiency of the fire safety subsystem under the test mode in comparison with the control mode is most evident at small values of the testing time. For example, under the control mode of the hydrogen storage and supply system with  $\omega_c=0.3$ , the reliability of operation is equal to 0.3 at  $\lambda_s \mu_s^{-1} = 0.01$ . Under the test mode at  $\omega_T=0.3$  and  $\lambda_s \mu_s^{-1} = \lambda_s \mu_T^{-1} = 0.01$  the reliability of functioning is 0.86.

With values of completeness of control (testing), which practically do not differ from unity:

- the effectiveness of the fire safety subsystem is the same for control and testing modes;

- for small testing times, the reliability value of the fire safety subsystem's operation under control and testing modes is close to unity. For example, for the control mode with  $\omega_c=0.9$  and  $\lambda_s \mu_s^{-1} = 0.01$  for the testing mode with  $\omega_T=0.9$  at  $\lambda_s \mu_s^{-1} = \lambda_s \mu_T^{-1} = 0.01$ , the reliability of functioning is  $0.9 \div 0.91$ .

**6. Discussion of the results of research on the effectiveness of the fire safety subsystem**

The central problem in the operation of hydrogen systems, in particular storage and supply systems, is ensuring their fire-explosion-safe condition at the required level. The solution to this problem relies on the fire safety subsystem, which is a structural element of the system of operation of hydrogen storage and supply systems. Control or testing of

the hydrogen storage and supply system using the fire safety subsystem reveals its fire-safe or fire-hazardous state. This process is accompanied by errors of the first and second kind. The effectiveness of the functioning of the fire safety subsystem is assessed by the degree of trust in the results obtained with its help – the reliability of functioning. The reliability of the functioning of the fire safety subsystem is interpreted as the conditional probability that this subsystem correctly recognizes the actual state of the hydrogen storage and supply system. The formalization of the functioning of the fire safety subsystem under control and testing modes is represented by graphs of its states. Such state graphs of the fire safety subsystem are matched with weight matrices, the elements of which are the corresponding intensities of transitions between states, taking into account the completeness of control or testing. The control efficiency of the hydrogen storage and supply system is determined by two additive components, which are the roots of the system of Kolmogorov equations. The system of these equations is given in matrix form, the main matrix of which has a size of  $4 \times 3$ . The effectiveness of the functioning of the fire safety subsystem under the control mode depends on two parameters – the completeness of control and the ratio that characterizes the level of fire safety of the hydrogen storage and supply system and the level of perfection of means of restoring its condition. It is advisable to increase the efficiency of the fire safety subsystem, first of all, by increasing the completeness of control over the hydrogen storage and supply system. The efficiency of hydrogen storage and supply system testing is determined by three additive components. These components are the roots of the system of Kolmogorov equations, the main matrix of which has a size of  $6 \times 5$ . The peculiarity of the testing mode is that its efficiency value does not reach unity even with the maximum completeness of the testing of the hydrogen storage and supply system. This is due to the final values of the parameter, which characterizes the level of fire safety of the hydrogen storage and supply system and the period of its testing. The second feature of the testing regime of the hydrogen storage and supply system is that the probability of its being in a state that corresponds to a fire-safe state can have a maximum value. This is possible, as follows from expression (2), if the square of the testing period is determined by three multiplicative values – the completeness of the test, the test time, and the parameter characterizing the level of fire safety of the hydrogen storage and supply system. The effectiveness of the fire safety subsystem in this case is determined by three parameters. Such parameters include completeness of testing, a parameter characterizing the level of fire safety of the system and the level of perfection of means of recovery, a parameter characterizing the level of fire safety of the system and the level of perfection of means of its testing. To compare the effectiveness of the fire safety subsystem in the control and testing modes, its quantitative evaluations were obtained, which are shown in Tables 1 and 2. This made it possible to identify additional features of the functioning of the fire safety subsystem of the hydrogen storage and supply system. In the case of control or testing completeness values that are significantly different from unity:

- the functioning of the fire safety subsystem under the test mode is more effective;

- the increase in the efficiency of the fire safety subsystem under the test mode relative to the control mode is most evident at small values of the testing time.

With values of control or testing completeness that differ little from unity:

- the effectiveness of the fire safety subsystem is the same for control and testing modes;
- for small testing times, the reliability value of the fire safety subsystem's operation in control and testing modes approaches unity.

The substantiation of the effectiveness of the functioning of the fire safety subsystem of the hydrogen storage and supply system is achieved due to:

- the use of a description of the operation of such a subsystem under control and testing modes in the form of state graphs – Fig. 1 and Fig. 3 with weight matrices (4) and (14);
- the use of Kolmogorov's equations in matrix form – expressions (8) to (10) and (17) to (19), the roots of which are components of expressions (12), (24), and (28), which describe the efficiency of subsystem functioning;
- the use of illustrative material – Fig. 2, 4, 5, as well as quantitative estimates given in Tables 1, 2.

Analysis of the dependences shown in Fig. 4, 5 reveals that the sensitivity of the effectiveness of the functioning of the fire safety subsystem, which is determined by expression (28), is more significant relative to the parameter that characterizes the level of perfection of the testing means.

The advantage of the given approach to increasing the level of safety in the operation of hydrogen storage and supply systems compared to known solutions [12, 14, 15] is that after detecting its fire-hazardous state, the system is restored to a fire-safe state. At the same time, an assessment of the effectiveness of this approach is provided. In [14], an increase in the level of safety is ensured by increasing the reliability of the hydrogen system as a result of softening the conditions of its operation. In [15], the reliability of the hydrogen system is increased due to the reservation of hydrogen tanks. In [12], the safety level of the hydrogen system is increased by increasing its reliability by improving the physical properties of the hydrogen storage tank. All these approaches are traditional and are not aimed at restoring the safe state of hydrogen systems.

The possibility of varying the modes of its operation should be attributed to the positive side when using the fire safety subsystem of the hydrogen storage and supply system.

Limitations in determining the efficiency of the functioning of the fire safety subsystem of the hydrogen storage and supply system are due to the assumption, according to which the evaluation of efficiency indicators is obtained.

The lack of research on the simultaneous control and testing of the state of the hydrogen storage and supply system can be attributed to the lack of the procedure for determining the effectiveness of the fire safety subsystem.

Further development of this area of research may be related to the determination of the effectiveness of the functioning of the fire safety subsystem of the hydrogen storage and supply system when combining control and testing modes.

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## 7. Conclusions

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1. It is shown that the degree of confidence in the results obtained with the help of the fire safety subsystem during control and testing of hydrogen storage and supply systems

is determined by the reliability of the functioning of such a subsystem. As the reliability of the functioning of the fire safety subsystem, which characterizes its effectiveness, the conditional probability that this subsystem correctly recognizes the actual state of the hydrogen storage and supply system is used. Under the control mode, the parameters of the storage and supply system are determined, which characterize the level of its fire danger directly in the regular mode of its operation. Under the test mode, a test effect is carried out on the hydrogen storage and supply system, based on the reaction to which the fire safety status of this system is determined.

2. A mathematical description of the functioning of this subsystem was built for the state control mode of the hydrogen storage and supply system using the fire safety subsystem. Such a mathematical notation is based on the use of graph theory, for which a weight matrix is presented, which includes a parameter characterizing the completeness of control and the intensity of transitions of the fire safety subsystem from one state to another. The effectiveness of the functioning of the fire safety subsystem is determined by the roots of the Kolmogorov equations, which are presented for the stationary regime in matrix form. The main matrix has a size of  $4 \times 3$ . An expression was obtained for the performance indicator of the fire safety subsystem, which was given in the form of a fractional-rational function. The arguments of this function are the completeness of control and the parameter that characterizes the level of fire danger of the hydrogen storage and supply system and the level of perfection of means of restoring its condition. It is noted that the reliability of the functioning of the fire safety subsystem asymptotically approaches 1 or the value of completeness of control when the value of completeness of control approaches 1 or at small values of the recovery time of the hydrogen storage and supply system, respectively.

3. Using graph theory, a mathematical model was built that describes the functioning of the fire safety subsystem under the mode of testing the state of the hydrogen storage and supply system. To determine the effectiveness of the functioning of such a subsystem, Kolmogorov's equations are used, which are represented in matrix form. The main matrix of this equation has a size of  $6 \times 5$ . An expression for the reliability of the functioning of the fire safety subsystem was obtained, which includes the completeness of testing, as well as the intensity of transitions of this subsystem from one state to another. It is shown that under the test mode the reliability of the functioning of the fire safety subsystem cannot reach 1. This is due to the fact that the ratio of the intensity of the transition of the hydrogen storage and supply system from the fire-safe state to the fire-hazardous state to the intensity of the period of its testing differs from 0. We obtained conditions under which it is achieved the maximum probability that the hydrogen storage and supply system will be in a fire-safe condition. In this case, the square of the testing period of such a system is determined by three multiplicative components: the completeness and time of testing, as well as the time of transition of the system from a fire-safe to a fire-hazardous state. For these conditions, an expression was obtained that describes the effectiveness of the fire safety subsystem. A comparison of the modes of operation of the fire safety subsystem shows that:

- with values of completeness of control (testing) that do not differ from 1, the effectiveness of functioning is invariant



with respect to the mode of functioning of the fire safety subsystem;

– with values of completeness of control (testing), which are significantly different from 1, the functioning of the fire safety subsystem under the testing mode is more effective.

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#### Conflicts of interest

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The authors declare that they have no conflicts of interest in relation to the current study, including financial,

personal, authorship, or any other, that could affect the study and the results reported in this paper.

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#### Data availability

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The data will be provided upon reasonable request.

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