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BUILDING A MODEL AND AN ALGORITHM FOR MODELING THE MOVEMENT OF PEOPLE CARRYING GOODS WHEN THEY ARE EVACUATED FROM PREMISES

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Evacuation is often the only way to save a person who is in a life-threatening situation. At present, evacuation software is used to simulate the movement of human flows, which does not always reflect the real processes of their movement. Therefore, it is a relevant task to build models for modeling the movement of human flows for different types of emergencies, different categories of human movement, and various spatial forms of their representation. Such a task arises when evacuating people from premises for various functional purposes.

During evacuation, people often carry some goods. When people move carrying some goods, their horizontal projection takes a more complex shape than an ellipse or circle considered in earlier studies. Moreover, in practice, there is often a task to model the movement of people taking into consideration the maximum permissible distances between them.

This paper reports the new quasi-phi functions of interaction between the ellipse and rectangle accounting for the maximum allowable distances between them. The proposed mathematical apparatus has made it possible to formalize the interaction between objects, thereby enabling the construction of a well-substantiated mathematical model, as well as the methods and algorithms for modeling the movement of people carrying some goods.

The possibility to simulate the movement of people with certain objects has shown taking into consideration the maximum permissible distances between them. A test example of the movement of people along four corridors was simulated, in each of which there were 28 people subsequently merging into one flow. Given the uniform distribution of three types of cargo: «backpacks», «suitcases», and «bags on wheels», the movement slowed down by about 4 %. When half of the evacuees had «bags on wheels» that can move away from people at arm's length, the slowdown was about 6 %

Keywords: heterogeneous flows of people, individual-flow movement, optimization by group of variables, nonlinear programming

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1. Introduction

The constant increase in the number of emergencies of the natural and man-made nature, including their scale, renders special relevance to the issue of protecting the population and territories from them. Natural disasters, terrorist acts, anthropogenic disasters lead to many deaths, most often because people could not leave the site of emergency in time.

Evacuation is often the only way to save a person who is in a life-threatening situation. To carry out operational and tactical activities to evacuate and rescue people from facilities, fire and rescue units are set up, which are typically formed according to the type of emergency that occurs in the building, that is, according to their professional skills. To help fire and rescue units is the aim of modern software complexes to manage evacuation from facilities whose main component is the programs that simulate human flows, which reflect the actual processes of their movement. In practice, there is often a task to model the movement of people taking into consideration the maximum permissible distances between them; the examples are family members or rescuers from one unit who move carrying some goods. When people move carrying some goods, their horizontal projection takes a more complex shape than an ellipse or circle, as is usually considered. Therefore, it is a relevant task to build models for modeling the movement of human flows for different types of emergencies, different categories of human movemen,t and various spatial forms of their representation.

2. Literature review and problem statement

As regards the issue under consideration, its main components are programs for modeling the movement of a human flow. There are no models of individual-flow movement of heterogeneous flows of people, which does not make it possible to build programs adequate to an actual flow [1]. The reason for this may be the lack of a mathematical apparatus for the analytical description of the conditions when people do not intersect. And, as a rule, people move carrying some goods; taking into consideration the maximum allowable distances between them complicates the spatial representation.

It should be noted that the task to model the movement of people at each specified discrete moment involves the configuration of the arrangement of objects under the specified constraints [2] and belongs to the class of problems of geometric design [3]. The main restrictions in this class are restrictions on the interaction of objects (conditions under which they do not intersect).

The first step in constructing an analytical description of the conditions under which objects within this class do not intersect was an approach reported in [4], which is based on the use of R-functions. R-functions were used to describe the conditions for the interaction of simple objects, such as circles; however, for some objects the R-functions took a complicated form. Further studies into the analytical description of interaction among objects are associated with the mathematical apparatus of dense arrangement functions (DAF) [5] and their hodograph (DAFH) [6], which made it possible to convert geometric information about the objects that are arranged into information about their possible dense placement.

Based on the DAFH, a methodology of sequential-single placement was developed to find approximations to local extrema in the problems of irregular arrangement of objects [6]. The variant of overcoming difficulties in finding a precise solution was the phi-function apparatus [7], which made it possible to describe appropriate problems in the form of nonlinear programming tasks and to apply accurate methods of solving. Phi functions for basic objects in two-dimensional [8] and three-dimensional spaces [9] were constructed.

Article [10] gives a thorough review of the literature on the tasks of arranging ellipses. However the authors failed to consider issues related to the continuous rotations of objects. Ae problem of the optimal arrangment of ellipses, allowing continuous rotation, was considered in [11]. Pseudo-normalized quasi-phi functions [12] are used to analytically describe the interaction of objects. In work [13], it was possible to simplify the analytical description of the relationship between ellipses (non-intersection and arrangement at the minimum permissible distance) using the quasi-phi function proposed by the authors.

The method of phi-functions, quasi-phi functions is recognized as the most powerful means of analytical modeling of relations between geometric objects in the world. That makes it possible to describe the optimization tasks of geometric design (cutting, packing, covering, and partitioning) in the form of nonlinear programming tasks and has a wide range of applications. Thus, the theory of optimization geometric design [3] is intended to solve a series of applied optimization problems of packing [14], parsing [15], coverage [16], traced coverage [17]. These tasks relate to devising energy- and resource-saving technologies in priority sectors of the national economy in the automation and modeling of the processes of arrangement of objects of different physical nature.

In the considered applied problem, the object of placement (movement) is a person. Work [13] shows that with the free category of movement, the most adequate model for mapping a human body onto a horizontal plane is an ellipse. The approach to modeling the free movement of people represented by ellipses is discussed in [13]. And, as a rule, people move carrying some goods. To model the movement of people, it is often necessary to take into consideration the maximum permissible distances between them. A general approach to the analytical description of the conditions under which objects do not intersect, taking into consideration the maximum permissible distances, is embedded in the theory of geometric design [3].

When modeling the movement of heterogeneous flows of evacuated people carrying some goods, their spatial shape, and, accordingly, horizontal projection, takes a more complex shape than an ellipse or circle. Therefore, an unresolved part of the problem in question is the construction of models and algorithms for modeling the movement of people carrying some goods during evacuation from buildings, based on an analytical description of the interaction of objects, taking into consideration their spatial shapes and the maximum allowable distances between them.

3. The aim and objectives of the study

The aim of this work is to build a model and algorithm for modeling the movement of people carrying some goods during their evacuation from buildings. That would make it possible to expand the range of practical tasks of modeling the movement of human flows, in particular, the movement of rescuers with appropriate equipment, patients with auxiliary aids, etc.

To accomplish the aim, the following tasks have been set:

– to build a mathematical model of the human body carrying some goods; to derive the analytical expressions of the conditions under which people do not intersect taking into consideration the maximum permissible distances between them;

– to build a mathematical model of the movement of people carrying some goods (objects of complex spatial shape) taking into consideration restrictions on the conditions under which they do not intersect and the maximum permissible distances between them; the coefficients of speed stability and maneuverability of movement;

– to develop an algorithm for finding a local-optimal solution to the problem on the rational movement of people carrying some goods on the horizontal path.

4. The study materials and methods

We studied the interaction of geometric objects, underlying the methods of modeling their arrangement considering the assigned restrictions and modeling the movement of people in the flow, by using a mathematical apparatus of phi functions, in particular quasi-phi functions [12].

Let us represent a projection of the human body for the problem of modeling human motion in the form of an object S_i. Each object S_i is assigned arrangement parameters $u_i=(\nu_i, \theta_i)$, where $\nu_i=(x_i, y_i)$ is the vector reflecting the object S_i relative to the stationary coordinate system, where θ_i is its rotation angle. Denote the object $S_i=S_i(0)$, rotated at angle θ_i and mapped onto the vector ν_i, through $S_i(u_i)$. In this case, an arbitrary point $p=p(0)$ of the object is mapped onto the point $p(\nu)=\nu+M(\theta_i)p^T(0)$, where $M(\theta_i)$ is the matrix of the space rotation operator at angle θ_i.

Consider the following relationships between the objects $S_i(u_i)$ and $S_j(u_j)$:

– intersection: $\text{int } S_i(u_i)\cap\text{int } S_j(u_j)\neq\varnothing$;

– contact: $\text{int } S_i(u_i)\cap\text{int } S_j(u_j)=\varnothing$ and $\text{fr } S_i(u_i)\cap\text{fr } S_j(u_j)\neq\varnothing$;

– non-intersection: $S_i(u_i)\cap S_j(u_j)=\varnothing$;

where int(•) – the interior of the topological, fr(•) – the set boundary(•).

The conditions under which two objects $S_i(u_i)$ and $S_j(u_j)$ do not intersect, where these objects are the above classes of objects, are to be built using the concept of their quasi-phi-function [12].

Definition 1: A quasi-phi function $\Phi'^{S_iS_j}(u_i,u_j,u')$ for the objects $S_i(u_i)$ and $S_j(u_j)$ is the fully defined function continuous for all variables, for which the function $\max\limits_{u'\in U\subset R^m}\Phi'^{S_iS_j}(u_i,u_j,u')$ is the phi-function of the objects $S_i(u_i)$ and $S_j(u_j)$ [10]. Here, u' is the vector of auxiliary variables belonging to some subset U of the space R^m [12].

The quasi-phi function proposed in article [12], unlike the phi function, depends not only on the placement parameters of the initial objects S_i and S_j but also on additional variables. The dimensionality of the space and its form depend on specific objects and arrangement conditions, modeling which involves a specific quasi-phi function.

An important characteristic of the quasi-phi function: if, for some u', $\Phi'^{S_iS_j}(u_i,u_j,u')\geq 0$, is satisfied, then $\text{int } S_i(u_i)\cap\text{int } S_j(u_j)=\varnothing$ [12].

Owing to the analytical description of the limitations in the problem of movement of people, in particular their interaction, it was possible to derive a mathematical model of the problem whose properties make it possible to use both existing methods of nonlinear programming and the optimization by groups of variables, which include the parameters of human arrangement.

5. Results of studying the modeling of the movement of people carrying some goods

5. 1. A mathematical model of the human body carrying some goods; the analytical expressions of conditions under which people do not intersect

A quasi-phi function for the ellipse and rectangle.

A quasi-phi function was modified to simulate the interaction conditions between the ellipse and rectangle. Let $E_i(u_i)$ be an ellipse with semi-axes a_i and b_i; $T_j(u_j)$ – a rectangle assigned by vertices $p_{ij}^j\,(i=1,2,3,4)$ with the arrangement parameters u_i, u_j, respectively. The normalized quasi-phi function for the ellipse $E_i(u_i)$ and the rectangle $T_j(u_j)$ takes the following form:

$$\Phi'^{E_iT_j}(u_i,u_j,u'_{ij})=\min\left\{\Phi'^{E_iD^+}(u_i,u'_1),\Phi'^{T_jD^-}(u_j,u'_2)\right\}, \qquad (1)$$

where $u'_1=t_1$, $u'_2=(c,d,r)$, $\Phi'^{E_iD^+}(u_i,u'_1)$ is the normalized quasi-phi-function for the ellipse $E_i(u_i)$ and semi-plane D^+, $\Phi'^{T_jD^-}(u_2,u')=\min\limits_{ij=1,2,3,4} f\left(-p_{ij}^j\right)$ is the normalized phi-function for the objects $T_j(u_j)$ and D^- [18], $f(x,y)=cx+dy+r=0$ $(c^2+d^2=1)$ – the equation of some straight line L_{ij}, which is the boundary of two semi-planes D^+ and D^-.

A quasi-phi function to simulate limiting the maximum allowed distance of an ellipse and rectangle.

To simulate limits on the maximum allowed distance ρ_{ij} for two objects $E_i(u_i)$ and $T_j(u_j)$, additional variables x_{ij}, y_{ij}, x_{ji}, y_{ji} are introduced, where $t_{ij}=(x_{ij}, y_{ij})$ and $t_{ji}=(x_{ji}, y_{ji})$ are the points belonging to the objects $E_i(u_i)$ and $T_j(u_j)$, respectively (Fig. 1).

If the distance $\rho_{ij}=\sqrt{\left(x_{ij}-x_{ji}\right)^2-\left(y_{ij}-y_{ji}\right)^2}$ is less than the maximum allowed distance δ_{ij} (Fig. 1, *a*) or, in some events, is met with a margin (Fig. 1, *b*), the limit for the maximum permissible distance between objects is met with a margin (objects may even intersect). Fig. 1, *c* shows the arrangement of objects at which the distance between the objects is δ_{ij}.

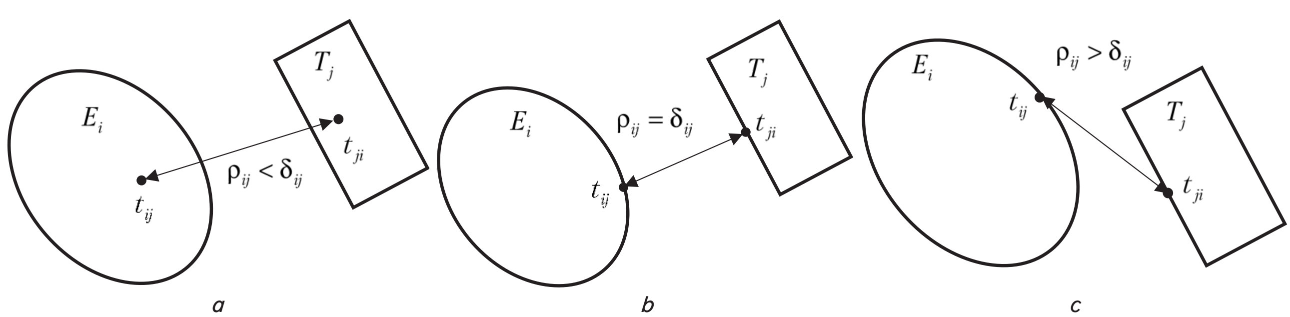


Fig. 1. Construction of a quasi-phi function for modeling restrictions on the maximum allowed distance for two objects: *a* – the distance ρ_{ij} is less than the maximum allowed distance δ_{ij}; *b* – the limit on the maximum allowed distance between objects is met with a margin; *c* – the distance between the objects is δ_{ij}

Thus, the quasi-phi function for modeling the limits on the maximum allowable distance for the two objects $E_i(u_i)$ and $T_j(u_j)$ can be stated as follows:

$$\Phi'^{E_iT_j}_{+}\left(u_i,u_j,u_{ij}\right)= \\ =\min\left\{\varphi^{E_it_{ij}}\left(u_i,t_{ij}\right),\varphi^{T_jt_{ji}}\left(u_j,t_{ji}\right),f\left(t_{ij},t_{ji}\right)\right\}, \quad (2)$$

where $\varphi^{E_it_{ij}}\left(u_i,t_{ij}\right)$, $\varphi^{T_jt_{ji}}\left(u_j,t_{ji}\right)$ are the membership functions of points $t_{ij}=(x_{ij}, y_{ij})$ and $t_{ji}=(x_{ji}, y_{ji})$ to the objects $E_i(u_i)$ and $T_j(u_j)$, respectively,

$$f\left(t_{ij},t_{ji}\right)=\delta^2-\rho_{ij}^2=\delta^2-\left(x_{ij}-x_{ji}\right)^2-\left(y_{ij}-y_{ij}\right)^2,\ \delta=\delta_{ij}.$$

The function of a point's membership with an object is a function that depends on the parameters of the object's location and the coordinates of the point. This function is positive when a point belongs to an object, equal to zero when a point is at the boundary of an object, and negative when a point does not belong to an object.

Note that the function $\Phi'^{E_iE_j}_{+}\left(u_i,u_j,u_{ij}^{+}\right)$ is a pseudo-normalized phi-function for modeling the maximum allowable distances between ellipses, and

$$\Phi'^{E_iE_j}_{+}\left(u_i,u_j,u_{ij}\right)\geq 0\Rightarrow dist\left(E_i\left(u_i\right),E_j\left(u_j\right)\right)\leq\delta_{ij},$$

where $dist(\upsilon_1, \upsilon_2)$ is the Euclid distance between points υ_1 and υ_2.

It should be noted that when modeling the movement of people there is no need to simulate restrictions with the accuracy that is needed in solving the problems of arrangement related to the industry. With a large dimensionality of problems, the cost of computational resources required to solve them significantly increases. Therefore, when modeling the movement of people in this work, it is proposed to use some approximation procedures. As an example, we give an approximate technique to construct conditions for the non-intersecting of the ellipse and rectangle, taking into consideration the maximum allowed distances.

Let the ellipse $E_i(u_i)$ be assigned with the semi-axes a_i, b_i, and the rectangle $T_j(u_j)$, given by vertices p_{ij}^{j} $(i=1, 2, 3, 4)$, respectively, with the arrangement parameters u_i, u_j. It should be noted that function (2) requires that four auxiliary variables should be introduced. We shall build an ellipse $E_i^{\delta}\left(u_i\right)$ with semi-axes $a_i+\delta$, $b_i+\delta$, (Fig. 2) (hereinafter, we denote through δ the maximum allowable distance δ between $E_i(u_i)$ and $T_j(u_j)$. It was proven in [6] that the ellipse $E_i^{\delta}\left(u_i\right)$ is the external δ-approximation of the ellipse $E_i(u_i)$.

Then the approximate quasi-phi-function for modeling the maximum permissible distances between the objects $E_i(u_i)$ and $T_j(u_j)$, depending on two additional variables, can be stated in the following form (Fig. 2, *a*):

$$\Phi'^{E_iT_j}_{+}\left(u_i,u_j,t_{ji}\right)=\min\left\{\varphi^{\hat{E}_it_{ji}}\left(u_i,t_{ji}\right),\varphi^{T_jt_{ji}}\left(u_j,t_{ji}\right)\right\}, \quad (3)$$

where $\varphi^{\hat{E}_it_{ji}}\left(u_i,t_{ji}\right)$ is the membership function of the point t_{ji} for the object $\hat{E}_i\left(u_i\right)=R^2\setminus\operatorname{int}E_i$, $\varphi^{T_jt_{ji}}\left(u_j,t_{ji}\right)$ is the membership function of the point t_j for the object $T_j(u_j)$. This function is built on the assumption that the rectangle (a cargo) faces a person with its longest side.

Let the point t_{ji} is not independent and is calculated from formula $t_{ji}=t_j^1+\lambda_{ji}\left(t_j^2-t_j^1\right)$ (Fig. 2, *b*). Then the approximate quasi-phi function for modeling the maximum permissible distances between the objects $E_i(u_i)$ and $T_j(u_j)$, depending on one additional variable λ_{ji}, can be stated as:

$$\Phi'^{E_iT_j}_{+}\left(u_i,u_j,\lambda_{ji}\right)= \\ =\min\left\{\varphi^{\hat{E}_it_{ji}}\left(u_i,t_{ji}\left(\lambda_{ji}\right)\right),\varphi^{T_jt_{ji}}\left(u_j,t_{ji}\left(\lambda_{ji}\right)\right)\right\}, \quad (4)$$

under the limitation $0\leq\lambda_{ji}\leq 1$.

It should be noted that functions (2) to (4) allow meeting the restrictions on the maximum allowed distances between a pair of objects. To satisfy the conditions of non-intersection, it is necessary to use the corresponding quasi-phi functions with their set of auxiliary variables, that is, function (2) requires that four points (Fig. 2, *c*) or eight auxiliary variables should be introduced.

The approximate phi function:

$$\Phi'^{E_iT_j}_{+}\left(u_i,u_j\right)= \\ =\min\left\{\begin{array}{l}\varphi^{\hat{E}_it_j^0}\left(u_i,t_j^0\left(t_j\right)\right),\\ \varphi^{E_it_j^1}\left(u_j,t_j^1\left(t_j\right)\right)-\varphi^{\hat{E}_it_j^0}\left(u_i,t_j^0\left(t_j\right)\right),\\ \varphi^{E_it_j^{21}}\left(u_j,t_j^2\left(t_j\right)\right)-\varphi^{\hat{E}_it_j^0}\left(u_i,t_j^0\left(t_j\right)\right),\\ \varphi^{E_it_j^3}\left(u_j,t_j^3\left(t_j\right)\right)-\varphi^{\hat{E}_it_j^0}\left(u_i,t_j^0\left(t_j\right)\right)\end{array}\right\}, \quad (5)$$

serves at the same time to describe the conditions under which objects do not intersect and the restrictions on maximum allowed distances. Restrictions on the use of such a function imply that the overall dimensions of the rectangle should be smaller than the overall dimensions of the ellipse.

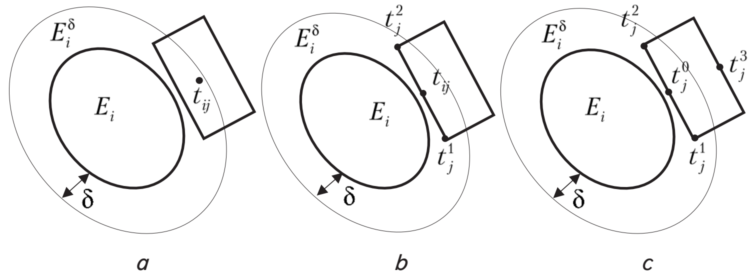


Fig. 2. The construction of approximate quasi-phi functions and a phi function for modeling restrictions on the maximum allowed distance for two objects: *a* – the quasi-phi function depends on additional variables $t_{ji}=(x_{ji}, y_{ji})$; *b* – the quasi-phi function depends on one additional variable λ_{ji}; *c* – the phi function requires that four points should be introduced (eight auxiliary variables)

Thus, the expansion of the analytical description of the conditions for object interaction by taking into consideration the maximum permissible distances makes it possible to expand the range of practical problems being solved. In particular, it has become possible to simulate the movement of people carrying some goods with the maximum distance between them. Examples are rescuers with appropriate equipment, people with auxiliary means of movement, taking into consideration their distribution by groups.

5. 2. Building a mathematical model of the movement of people carrying some goods under the specified restrictions

The motion path is divided into Ω_m sections, numbered, respectively, 1.2, ..., m, and limited by the delimiters $A_1, A_2, ..., A_{m+1}$. Each section is characterized by its specific law that forms the main direction of movement. The sections with rectilinear movements have been considered. In these sections, the movement from the point being analyzed is given as a vector connecting this point to a point at the corresponding delimiter (taking into consideration the homothety coefficient). Assume that at the k-th iteration (in the predefined time interval Δt, for example, 1 s), the evacuation section Ω_m hosts N_k people carrying some goods. Denote a set of people in the form of the set $S'=\{E_1,E_2, ..., E_i, ..., E_{Nk}\}$, and, accordingly, their cargoes – $S''=\{T_1,T_2, ..., T_j, ..., T_{Nk}\}$. A set of the consecutive pairs of objects (ellipses and rectangles) is denoted $H=\{H_1,H_2, ..., H_i, ..., H_{Nk}\}=\{E_i,T_j\}_{2Nk}$. The object H_i is represented by a combination of an ellipse (the horizontal projection of a human body) and a rectangle (a cargo). The ellipse is set by the size of the semi-axes (a_i, b_i), has the arrangement parameters $u_i=(x_i, y_i, \theta_i)$, $i=1, 2, ..., N_k$, where (x_i, y_i) are the coordinates of the origin of the local coordinate system (current point), θ_i – the angle of rotation of the i-th ellipse E_i. The rectangle T_j is given by vertices p_{ij}^j $(i=1, 2, 3, 4)$, respectively, with the parameters (x_j, y_j) and θ_i. Restrictions on the overall dimensions of the rectangle are considered, they should be smaller than the overall dimensions of the ellipse, and at the maximum permissible distance between the ellipse and rectangle δ_{ij}. We also note that the large half-axis of the ellipse is perpendicular to the direction of movement, and the angle of rotation θ_i of the ellipse E_i is determined between the perpendicular to the large half-axis and the vector of the main direction of movement. The object E_i is also assigned the characteristics of speed ν_{ki} (meters per second) and maneuverability m_{ki}, $m_{ki}<1$ (meters). For each current point with the coordinates $g_i(x_i, y_i)$ (the coordinates of a person's location), we determine the vector of speed $\vec{v}_i\left(x_{i,k}, y_{i,k}\right)$. The speed vector depends on the local flow velocity and is determined according to experimental data [19]. This value is average. However, each person, based on his/her physical capabilities, has a specific movement speed, which is adjusted by the speed stability factor $k_{1i}\in[0, 1]$. The speed stability sets the readiness of an individual to temporarily accelerate, that is, to «overtake» to occupy a better position to move. Consider the maneuverability of each person, that is, the ability to deviate from the main direction of movement. The average statistical maneuverability is also adjusted for each person by the maneuverability coefficient $k_{2i}\in[0, 1]$. Thus, each object H_i is characterized by speed $k_{1i}\left|\vec{v}_i\right|$ (meters per second) and maneuverability – $k_{2i}m_i\le z_i\le k_{2i}m_i$, $|m_i|\le 1$ (meters). Then the mathematical model of the subtask at the k-th iteration can be stated in the form of a search for the maximum aggregate movement of people in the evacuation area [13], taking into consideration the conditions under which complex objects do not intersect (and taking into consideration the maximum permissible distances between a person and cargo), the conditions for their positioning in section Ω_m, the coefficients of stability of speed and maneuverability. That is, it is required to find:

$$F\left(u^*\right)=\max_{u\in W_k\subset R^n} F\left(u\right), \tag{6}$$

$$W_k=\left\{u\in R^n:\gamma_{ij}\ge 0;\ \gamma_i\ge\rho_i;\ T_i\ge 0;\ i<j\in I_{N_k}\right\},$$

where

$$u=\begin{pmatrix}\Delta t_1, z_1, u_1, \Delta t_2, z_2, u_2, ..., \Delta t_{2N_k}, z_{2Nk}, u_{11}^+, ..., \\ u_{N_kN_k}^+, u_{12}', ..., u_{ij}', ..., u_{2N_k-1,2N_k}'\end{pmatrix},$$

$$n=14N_k+5N_k(2N_k-1),$$

$$F\left(u\right)=\Delta t\sum_{i=1}^{N_k}\Delta t_i\left|\vec{v}_i\right|, \tag{7}$$

$$\gamma_{ij}\ge 0:\Phi_{+}''^{E_iT_j}\left(u_i, u_j, u_{ij}^+\right)\ge 0,\ i, j=1, ..., N_k\in I_{N_k},$$

$$u_{ij}^+=\left(t_{ij}, t_{ji}\right)=\left(x_{ij}', y_{ij}', x_{ji}', y_{ji}'\right), \tag{8}$$

$$\Phi'^{S_iS_j}\left(u_i, u_j, u_{ij}'\right)\ge 0,$$

$$u_{ij}'=\left(t_i, c_{ij}, d_{ij}, r_{ij}\right)=\left(x_{ij}', y_{ij}', , c_{ij}, d_{ij}, r_{ij}\right),$$

$$i>j=1, ..., N_k;\ S_i, S_J\in\left\{E_i, T_j\right\}_{2N_k}\subset H. \tag{9}$$

$$\gamma_i\ge\rho_i;\ \Phi^{H_i\Omega_m^*}\left(u_i\right)\ge\rho_i, i=1,2,\ ..., N_k\in I_{N_k}, \tag{10}$$

$$T_i\ge 0:\begin{cases}0\le\Delta t_i\le 1, \\ -k_{2i}m_i\le z_i\le k_{2i}m_i,\ i\in I_{N_k},\end{cases} \tag{11}$$

$$x_i=x_i+k_{1i}v_{i,x}\Delta t_i\Delta t-z_id_i^y\Delta t_i,$$

$$y_i=y_i+k_{1i}v_{i,y}\Delta t_i\Delta t+z_id_i^x\Delta t_i,$$

$$\theta_i=\theta_i+\Delta_i\Delta t_i,$$

where Δt_i is the relative step in the time of movement of the i-th person; $\Delta_i=\hat{\theta}_i-\theta_i$, $\hat{\theta}_i$ is the angle of ellipse rotation at point $\left(x_i+v_{i,x}\Delta t_i\Delta t, y_i+v_{i,y}\Delta t_i\Delta t\right)$; quasi-*phi*-functions (8), which can be described approximately in the form (3) to (5); (9) are the conditions of mutual non-intersection of objects that can be represented by expression (1); (10) are the arrangement conditions in section Ω_m that are described by the *phi* functions considered in [13]; (11) are the restrictions on the interval of time and the maneuverability of movement.

The problems of conditional optimization (6) to (11) refer to the NP-complex category. The region of acceptable solutions W_k has a complex structure: it is, generally speaking, an unrelated set, each component of which is multiconnected.

5. 3. An algorithm for finding a locally optimal solution to the problem

Based on the properties of the mathematical model, an algorithm has been proposed for modeling the movement of people carrying some goods involving the optimization by groups of variables, which include parameters for placing people.

Algorithm:

Step 1. The evacuation zone is represented as a graph. Edges are the segments of corridors; vertices are the intersections and points of «gluing» the segments (corridors).

Step 2. Determine, for each segment of the evacuation zone, the direction of preferential movement.

Step 3. In the evacuation zone, build a grid with a small step to determine the density of the flow.

Step 4. People represented by ellipses are sorted according to increasing the distance to the exit.

Step 5. Determine, for each person, for the coordinates of the center position and the angle of rotation, the speed of movement, which is adjusted according to the speed stability factor.

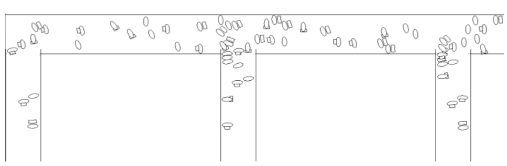
Step 6. The angle of rotation of each object is determined according to the average maneuverability of the individual and the individual coefficient of maneuverability.

Step 7. Determine the region of permissible solutions, which is described by the conditions of the non-intersection of complex objects (an ellipse and a rectangle) taking into consideration the maximum allowable distances between their components and the conditions for their arrangement in the segment.

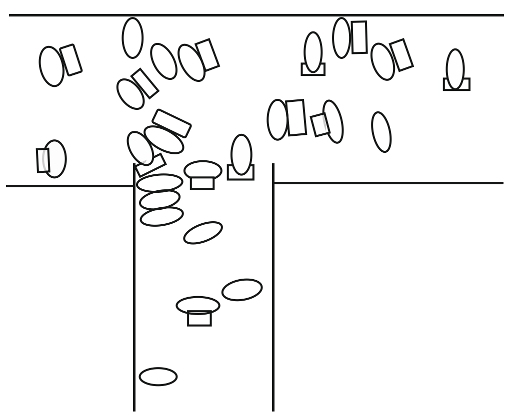
Step 8. Compute the rational parameters for placing complex objects in the region of acceptable solutions, which allow the maximum cumulative movement of people carrying some goods per selected time unit.

As an example, we have simulated the movement of people carrying some goods using a set of programs written in the C++ language in the Visual C programming environment.

Fig. 3 shows a fragment of people evacuation along three corridors implying merging the flow of people in the main corridor. People are approximated by ellipses whose sizes are given by the sizes of their half-axes. The large half-axis of each ellipse is perpendicular to the main direction of movement. It is possible to rotate the ellipses within the maneuverability angles.



a



b

Fig. 3. Fragments of arranging people that carry some goods at some fixed time of movement: *a* – when people move along three corridors with access to the main one; *b* – in the neighborhood of the second corridor

Some people have cargo, which is in «backpacks» and «suitcases», represented by rectangles that are given by length and width. «Suitcases» can partially intersect with ellipses in a person's shoulder area, as they are placed below. Some people move the cargo with the help of «bags on wheels»; at the same time, bags specified by rectangles can be removed from the person at the distance of the «stretched hand», which sets the maximum permissible distance between the person and the «bag». In this case, «backpacks» and «bags on wheels» can intersect, as they are at different heights.

The speed of movement of people varies depending on the change in the local flow velocity. The case has been considered where the average speed of movement of a person with cargo coincides with the average speed of movement without cargo (only the influence of geometric restrictions is taken into consideration).

We have simulated the flow of people moving along four corridors, each of which hosts 28 people, subsequently merging into one (a test example from [19]). The movement time when people move without cargo is 98 s [13]. In the presence of «bags on wheels», half of the evacuees had a slowdown of about 6 %, and with a uniform distribution of three types of cargo – by about 4 %.

6. Discussion of results from the implementation of the constructed mathematical models and their practical use

In the problems of geometric design, in particular, when modeling the movement of heterogeneous flows of people, one of the key points is an analytical description of the conditions for their mutual non-intersection. The quasi-phi functions for the conditions of ellipse and rectangle non-intersection (1) have been modified in this work, based on which a quasi-phi function for the ellipse with a rectangle (Fig. 1) has been constructed, taking into consideration the maximum allowable distances between them; illustrated by expression (2). Expression (2), which is the development of the quasi-phi-function (1), is the basis for building a model to model the movement of people carrying some goods.

It should be noted that when modeling the movement of people involving a large dimensionality of problems arising in practice, there is no need to model restrictions with the accuracy that is required when solving the problems of placing geometric objects. Therefore, for the problems of modeling the movement of people, in this work we have additionally proposed to use some approximating procedures. As an example, this article provides an approximate technique to build conditions under which the ellipse and rectangle do not intersect, taking into consideration the maximum allowed distances (3), (4). It should be noted that functions (2) to (4) only allow for meeting the restrictions on the maximum allowed distances between a pair of objects. To satisfy the conditions of non-intersection, these conditions are complemented by the approximate conditions of object non-intersection (5).

The devised mathematical apparatus is the justified basis for building a mathematical model of the maximum movement of people carrying some goods at each selected time interval; expressions (6) to (11). We have investigated the properties of the model, the main of which, the problem of conditional optimization (6) to (11), is an NP-complex task of nonlinear programming. The region of permissible solutions to problem (8) to (11) has a complex structure: it is, generally speaking, an unrelated set, each component of which is multiconnected. The properties of the model have allowed us to modify the algorithm for modeling the movement of people carrying some goods, which is the basis of software to simulate their movement.

The adequacy of the mathematical model of the individual-flow movement of people, represented as ellipses, is shown in work [13]. The deviation between computer simulation results [13] and the experimental data reported in [19] did not exceed 5 %. The model considered in this work accounts for additional restrictions on the movement of people carrying some goods that cannot move away from a person (ellipse) at the maximum permissible distance.

For the analytical description of these limitations, the phi-function apparatus has been advanced, in particular, quasi-phi functions, which showed its effectiveness and adequacy in solving multi-numerical applied optimization problems for the arrangement of three-dimensional bodies and two-dimensional objects (cutting, packing, and coverage) [3–17].

The impact of cargo size at the time of evacuation was investigated. An example of motion modeling involving 128 people was considered. The average evacuation time without cargo is about one and a half minutes (98.253 seconds) for ten launches of the program. It turned out that the presence of cargo: «backpacks» with a width of about 40 cm, and «suitcases» with a width of about 50 cm did not have a significant impact on the evacuation speed (slowing down less than 2 %). In the presence of «bags on wheels», half of the evacuees saw a slowdown of about 6 %, and with a uniform distribution of three types of cargo – about 4 %.Thus, the simulation of the movement of people carrying some goods has shown that the greatest impact on the time of evacuation is exerted by the movement in «bottlenecks» of a «bag on wheels», both because of the size, larger than in hand luggage, and because of the larger additional area between a person and a bag.

This paper has considered the restrictions on the number of components of displaced objects, between which the maximum permissible distances are maintained (only two objects are considered: an ellipse and a rectangle), which is not fundamental. Therefore, further research may consider the tasks of modeling the movement of people who can be described by complex objects with a larger number of components and which form groups, taking into consideration the predefined maximum permissible distances between their members.

As further practical applications, it is possible to consider the evacuation of people that carry some goods from hospitals, nursing homes, moving robots, etc.

7. Conclusions

1. A mathematical model of the body of a person that carries some goods has been built. The model is a combination of an ellipse and a rectangle. The ellipse approximates the human body: the adequacy of such a representation for the tasks of modeling the movement of people is shown, both in our earlier works and in the experimental studies by other researchers. A cargo is represented as a rectangle. The shape of cargo is not fundamental for investigation: the mathematical apparatus for describing the conditions of non-intersection makes it possible to consider objects of a more complex shape. We have derived analytical expressions for a pair of objects (ellipse and rectangle) taking into consideration the maximum allowable distances between them. The proposed mathematical apparatus makes it possible to expand the range of practical tasks being solved, both in the evacuation problem class and in geometric modeling theory. It has become possible to simulate the movement of people carrying some goods with the maximum allowable distance between them for example, a rescuer and appropriate equipment), and without it, people with auxiliary means of movement, and taking into consideration their distribution by groups, etc.

2. A mathematical model of the movement of people carrying some goods has been built considering the specified restrictions. Analytical expressions for the conditions of non-intersection of objects and the coefficients of stability of speed and maneuverability act as restrictions. The speed of movement is determined, as a rule, depending on the density of the flow, which is established experimentally. It should be noted that the speed and maneuverability are adjusted by the speed and maneuverability coefficients. These coefficients make it possible to take into consideration the individual capabilities of each person, in contrast to averaged data. The above constraints form the region of acceptable solutions to the problem. The region of acceptable solutions has a complex structure: this is, generally speaking, an unrelated set, each component of which is multiconnected; the problem refers to the NP-complex tasks of nonlinear programming.

3. Based on the properties of the mathematical model, an algorithm of modeling the movement of the flow of people has been modified. The modification involves constructing new algorithms to prevent the intersection of the ellipse and rectangle, taking into consideration the maximum allowable distances. On the basis of these algorithms, a technique of the consecutive movement of people taking into account coefficients of stability of speed and maneuverability that take into consideration the individual characteristics of people has been devised. The adequacy of the model has been shown by comparing the results of computer simulations of ellipse displacement with the experimental data reported in the literature (the deviation does not exceed 5 %). As regards the objects considered in this work, the efficiency and adequacy of the mathematical apparatus of interaction of objects has been proven by numerous publications on geometric design, which includes the problem of modeling the movement of people.

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