Development the methods of optimum placement undirected planar objects with piecewise non-linear boundaries in the multiply area

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Abstract. In this paper the statement of the problem is formulated and the mathematical model of optimization the placement of the undirected planar geometrical objects with piecewise non-linear boundaries in the multiply area is developed. It is shown the geometrical interpretation and derived the estimate of the number of restrictions in the model. On the basis of a mathematical model for finding the global extremum of the objective function was proposed modified method of branches and boundaries. It is also shown the solutions tree that takes into account the problems of optimal placement of undirected planar geometrical object with piecewise non-linear boundaries in the multiply area, and received the complexity of this method. For locally optimal solutions of the problem modified simulated annealing method has been developed. Thus the analytical expressions for the function of energy system were received, the function, that describes the decrease of temperature over time, function that forms a new state of system. The method of formation the new state of the system was investigated in more detail, which is based on a random permutation of numbers the pair of the objects, it is also based on a consistent placement of objects according to reshuffle their numbers and determining the probability of transition to a new state. It is shown the example of determining permissible points of placement the local coordinate system of the specific geometrical object. The conclusion is that to solve practical optimization problems of placement of the undirected planar geometrical objects with piecewise non-linear boundaries in the multiply area should be used the modified simulated annealing method.

Key words: optimal placement, the object with piecewise linear boundary, multiply area, mathematical model, branch and bound method, the method of simulated annealing.

INTRODUCTION

At present, for modeling of real technological and economic processes in the creation of technical systems that are associated with the processing of complex geometric information, there is a necessity of using effective methods of geometric and computer modeling. Important place among the tasks associated with processing of geometrical data, take the problems of geometric design optimization (optimization of placement, coverage, splitting the objects of the best lines). These problems arise in such sectors as light and heavy industry (designing of the cutting card), energetics, engineering, construction and so on.

Among the class of problems of optimization of the geometric design the problem of optimal placement of geometric objects is one of the most studied tasks. There are numerous methods for optimizing the placement of flat objects in simply, multiply and unconnected sectors, the methods of optimization the placement of three-dimensional objects and more. The peculiarity of the problems of optimal placement is that when the decision is necessary to satisfy the basic requirements for a mutual non-intersection of geometric objects that are placed in a given area, as well as supply of geometric objects of placement in the area. For analytical description of these requirements the constructive device of Φ-functions was developed in the professor Y. Stoyan's scientific school. Despite the fact that the wording of the main requirements is the same for all the problems of optimal placement of geometric objects, their analytical and geometrical interpretation appearance will vary considerably depending on the shape of geometric objects that are placed and area of placement.

One of the promising sector of researches is the geometrical modeling optimization of placement of the undirected planar geometry with piecewise non-linear boundaries in a given area. The peculiarity of these problems is as follows: first, the presence of fragments of nonlinear boundary leads to the necessity of considering the nonlinear constraints for parameters of placement the related objects; secondly, geometric objects are undirected, which increases the dimension of these problems by increasing of parameters of placement the objects. Thus, consideration of these features makes the development of new models and methods of geometrical modeling of optimization the placement of the undirected planar objects with piecewise non-linear boundaries in the given areas.

THE ANALYSIS OF RECENT RESEARCHES AND PUBLICATIONS

The development of mathematical models and methods of solving the problems of optimization of geometric design is dedicated to a sufficiently large number of works. Thus, the analytical presentation of boundaries of geometrical objects is devoted such works as [1-3]. In publications [4, 5] the mathematical models and methods of optimization design are studied. The tasks of optimal routes are dedicated in [6, 7]. The problems of optimal coverage and breaking were studied, for example, in the publications [8-11].

The modeling of physical processes in different sectors are dedicated the works [12, 13]. In the work [14], the determination of the optimal structure of the territorial system of technological safety is described.

As for optimization of problems the placement oriented flat geometry, they were considered, for example, in the works [15-20]. Modeling of optimal placement undirected objects are, for example, in [21, 22].

In the work [23] the model and method of placement optimization oriented flat geometry of piecewise with piecewise non-linear borders is devoted. In this publication [24] the issue of geometric information in optimization problems placement of undirected planar geometry with piecewise non-linear boundaries was studied. The work [25] is devoted to development of mathematical models of optimization the placement of undirected planar objects with piecewise non-linear boundaries and researching its features.

OBJECTIVES

In this work it is necessary to develop the methods of minimizing the objective function in the problem of optimal placement of undirected planar geometry with piecewise non-linear boundaries, based on the method of branches and boundaries and the method of simulated annealing. It is also necessary to conduct a comparative analysis of these methods to justify their further use for solving practical problems.

THE MAIN RESULTS OF THE RESEARCH

Let us consider the setting of optimal allocation undirected planar geometrical objects with piecewise non-linear boundaries in the multiply sector.

Let the two-dimensional space objects accommodation

$$S_i\left(x_i, y_i, \theta_i\right),\ i=1,2,\ldots,N,$$

with piecewise non-linear boundaries are specified. These objects are undirected and set its sequence of vertices

$$\left\{v_{i1}, v_{i2}, \ldots, v_{im_i}\right\},\ v_{id}=\left(x_{id}\left(\theta_i\right), y_{id}\left(\theta_i\right)\right),$$
$$d=1,2,\ldots,m_i,$$

in the local coordinate system and the numbering of tops is made counterclockwise. Each pair of vertices $\left(v_{id}, v_{id+1}\right)$ is connected with curve fragment of 2-nd order:

$$\begin{aligned}&a_{i,dd+1,1}\left(\theta_i\right)x_i^2+a_{i,dd+1,2}\left(\theta_i\right)x_i y_i+\\&+a_{i,dd+1,3}\left(\theta_i\right)y_i^2+a_{i,dd+1,4}\left(\theta_i\right)x_i+\\&+a_{i,dd+1,5}\left(\theta_i\right)y_i+a_{i,dd+1,6}\left(\theta_i\right)=0,\end{aligned}\qquad(1)$$

where: $a_{i,dd+1,c}\left(\theta_i\right)$, $c=1,\ldots,6$ – quadratic form parameters that are describing the fragment boundaries between vertices v_{id} and v_{id+1} of object $S_i\left(x_i, y_i, \theta_i\right)$.

The placement area $S_0(l,b)$ is a rectangle that is specified in the global coordinate system, and its length is variable (Fig. 1). This rectangle sector includes the sectors of prohibitions $S_{0,r}\left(x_{0,r}, y_{0,r}\right)$, $r=1,2,\ldots,N_R$ (e.g. defects of material or placing objects that are at fixed locations) that can be given to similar accommodations to the objects of replacement, but the numbers of vertices is performed clockwise.

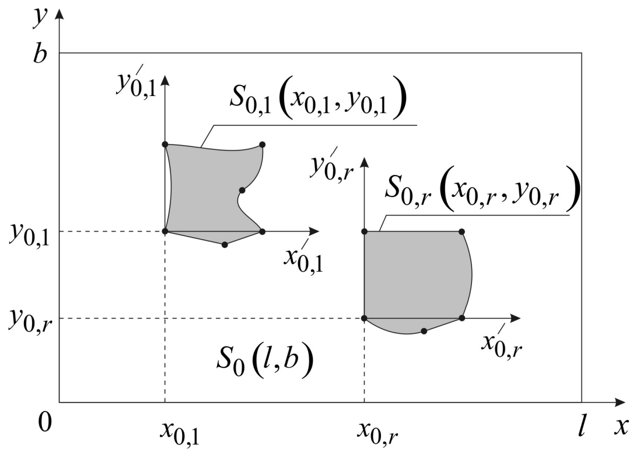

Fig. 1. Multiply placement area

It‘s necessary to place objects $S_i\left(x_i, y_i, \theta_i\right)$, $i=1,\ldots,N$, in the multiply area $S_0(l,b)$ so that length l was minimal and thus the restrictions must be enforced on:

– mutual non-intersection of the objects $S_i\left(x_i, y_i, \theta_i\right)$ and $S_j\left(x_j, y_j, \theta_j\right)$, $i=1,\ldots,N$, $j=i+1,\ldots,N$,

– non-intersection of the objects $S_i\left(x_i, y_i, \theta_i\right)$ and areas of prohibition $S_{0,r}\left(x_{0,r}, y_{0,r}\right)$, $r=1,2,\ldots,N_R$,

– objects belonging $S_i\left(x_i, y_i, \theta_i\right)$ in the area $S_0(l,b)$.

Formulate a mathematical model of optimal placement of undirected planar objects with piecewise non-linear boundaries in the multiply area:

$$\min_W l\left(x_1, y_1, \theta_1, \ldots, x_N, y_N, \theta_N\right)\qquad(2)$$

where W:

$$\Phi_{ij}\left(x_i, y_i, \theta_i, x_j, y_j, \theta_j\right)\geq 0,\ i=1,\ldots,N-1,$$
$$j=i+1,\ldots,N;\qquad(3)$$

$$\Phi_{kr}\left(x_k, y_k, \theta_k, x_{0,r}, y_{0,r}\right)\geq 0,\ k=1,\ldots,N,$$
$$r=1,\ldots,N_R;\qquad(4)$$

$$\Phi_{icS_0}\left(x_i, y_i, \theta_i, 0, 0\right)\geq 0,\ i=1,\ldots,N.\qquad(5)$$

In the model (2)÷(5) the formulae (2) is the objective function of the problem; restriction (3) – is a condition of the mutual non-intersection of the objects; the restriction (4) – is a condition of the non-intersection the objects of placement and prohibition areas; the restriction (5) – is a condition of belonging the objects to the placement area, where cS_0 – additions of S_0 to the two-dimensional space.

It should be noted that the conditions of (3) and (4) are, in general, non-linear, and the conditions (5) – linear. All restrictions analytically are submitted by Φ-functions [4], and their total number is $C_N^2 + N(N_R + 1)$. To formalize the restrictions (3)÷(5) the method given in [26] is used.

Geometric interpretation of condition (3) for fixed θ_i and θ_j is shown in Fig. 2

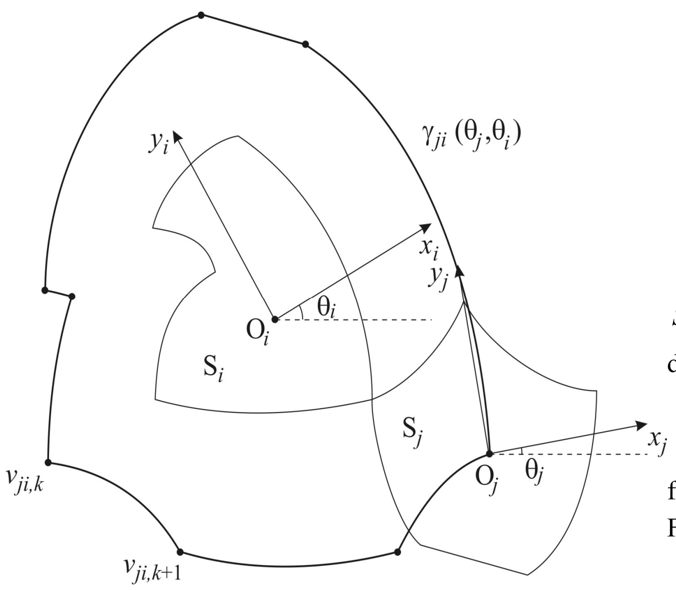

Fig. 2. Geometric interpretation of condition (3)

In general, the condition of the contact the objects $S_j(x_j, y_j, \theta_j)$ and $S_i(x_i, y_i, \theta_i)$, has the form:

$$\begin{aligned} &a_{ji,kk+1,1}(\theta_j,\theta_i)x^2 + a_{ji,kk+1,2}(\theta_j,\theta_i)xy + \\ &+a_{ji,kk+1,3}(\theta_j,\theta_i)y^2 + a_{ji,kk+1,4}(\theta_j,\theta_i)x + \\ &+a_{ji,kk+1,5}(\theta_j,\theta_i)y + a_{ji,kk+1,6}(\theta_j,\theta_i) = 0; \end{aligned} \qquad (6)$$

where: $a_{ji,kk+1,t}(\theta_j,\theta_i)$, $t = 1,\ldots,6$ – are quadratic form parameters describing the piece between k and $(k+1)$ vertices of the contact contour $\gamma_{ji}(\theta_j,\theta_i)$ of the objects $S_j(x_j, y_j, \theta_j)$ and $S_i(x_i, y_i, \theta_i)$, $j = 1,\ldots,N$, $i = 1,\ldots,N$, $j \neq i$.

It should be noted that the geometrical interpretation of the condition (4) is similar to (3) with one exception: the numbering of the vertices contact contour is made counterclockwise.

Fig. 3 shows a geometric interpretation of the condition (5) for fixed θ_i (placement area for illustration of the condition is simply connected).

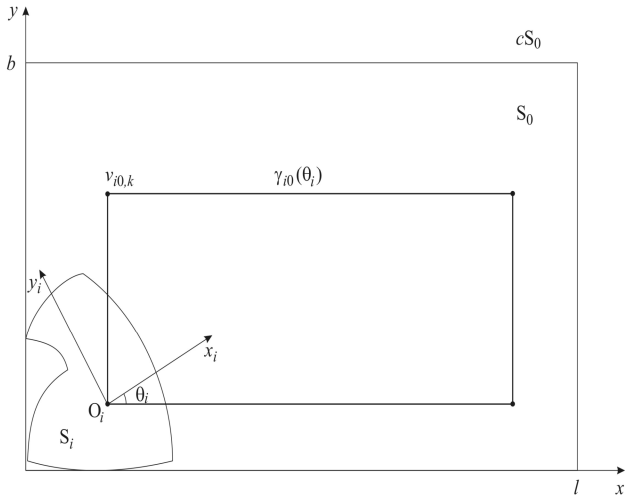

Fig. 3. Geometric interpretation conditions (5)

Thus, the condition of the contact the object $S_i(x_i, y_i, \theta_i)$ and addition of the area S_0 to the two-dimensional space is written as:

$$a_{i0,k}(\theta_i)x + b_{i0,k}(\theta_i)y + c_{i0,k}(\theta_i) = 0;\ \ k = 1,\ldots,4\,. \quad (7)$$

To search of global extremum of the objective function (2) construct a tree of solutions that is shown in Fig. 4.

Each level of the tree corresponds the independent variable of this problem, whose number is $3N+1$. At the appropriate level solutions trees are written in pieces of contact contours of geometric objects. For example, for the level of tree, that is corresponding the independent variable x_1, are written in equation, that consist of this variable.

The complete change of tree branches that is shown in Fig. 4, will determine the global extremum of the objective function (2). For permissible values of the objective function of the problem is necessary to solve a system of $3N+1$ equations (both linear and nonlinear), and a fragment of contour $\gamma_{ji,k}(\theta_j,\theta_i)$ is described by the equation of type (6) and contour $\gamma_{i0,k}(\theta_i)$ is described by the equation of type (7). Unacceptable branches of the solutions tree are cut off by means of the relevant rules.

$$x_1 : \gamma_{12,1}(\theta_1,\theta_2)\ldots\gamma_{12,n_{\gamma_{12}}}(\theta_1,\theta_2)\ldots\gamma_{1N,n_{\gamma_{1N}}}(\theta_1,\theta_N)\ldots\gamma_{1N_R,n_{\gamma_{1N_R}}}(\theta_1)\ldots\gamma_{10,4}(\theta_1)$$

$$y_1 : \gamma_{12,1}(\theta_1,\theta_2)\ldots\gamma_{12,n_{\gamma_{12}}}(\theta_1,\theta_2)\ldots\gamma_{1N,n_{\gamma_{1N}}}(\theta_1,\theta_N)\ldots\gamma_{1N_R,n_{\gamma_{1N_R}}}(\theta_1)\ldots\gamma_{10,4}(\theta_1)$$

$$\theta_1 : \gamma_{12,1}(\theta_1,\theta_2)\ldots\gamma_{12,n_{\gamma_{12}}}(\theta_1,\theta_2)\ldots\gamma_{1N,n_{\gamma_{1N}}}(\theta_1,\theta_N)\ldots\gamma_{1N_R,n_{\gamma_{1N_R}}}(\theta_1)\ldots\gamma_{10,4}(\theta_1)$$

$$\vdots$$

$$x_N : \gamma_{N1,1}(\theta_N,\theta_1)\ldots\gamma_{NN-1,n_{\gamma_{NN-1}}}(\theta_N,\theta_{N-1})\ldots\gamma_{NN_R,n_{\gamma_{NN_R}}}(\theta_N)\ldots\gamma_{N0,4}(\theta_N)$$

$$y_N : \gamma_{N1,1}(\theta_N,\theta_1)\ldots\gamma_{NN-1,n_{\gamma_{NN-1}}}(\theta_N,\theta_{N-1})\ldots\gamma_{NN_R,n_{\gamma_{NN_R}}}(\theta_N)\ldots\gamma_{N0,4}(\theta_N)$$

$$\theta_N : \gamma_{N1,1}(\theta_N,\theta_1)\ldots\gamma_{NN-1,n_{\gamma_{NN-1}}}(\theta_N,\theta_{N-1})\ldots\gamma_{NN_R,n_{\gamma_{NN_R}}}(\theta_N)\ldots\gamma_{N0,4}(\theta_N)$$

$$l : \gamma_{10,4}(\theta_1)\ldots\gamma_{N0,4}(\theta_N)$$

Fig. 4. Solution tree

The upper bound of the complexity (the number of equations system to be solved to determine the parameters of appropriate placement of geometric objects) of the developed modified method of branches and borders is as follows:

$$O_1 = N\cdot\prod_{i=1}^{N}\left(\sum_{\substack{j=1,\\ j\neq i}}^{N} n_{\gamma_{ij}} + \sum_{r=1}^{N_R} n_{\gamma_{ir}} + 4\right)^3; \qquad (8)$$

where: N – is the number of placement objects; $n_{\gamma_{ij}}$ – is the number of contact contour pieces for objects $S_i(x_i, y_i, \theta_i)$ and $S_j(x_j, y_j, \theta_j)$; $n_{\gamma_{ir}}$ – is the number of contact contour pieces for objects $S_i(x_i, y_i, \theta_i)$ and $S_{0,r}(x_{0,r}, y_{0,r})$

The disadvantage of the modified branch and bound method is that the conclusion of the global extremum of the objective function (2) can be done only after a search of all the branches of the solutions tree, which is an excessive (8) number of. This leads to the fact that to apply this method for solving practical tasks is almost impossible. To overcome these drawbacks propose we suggest a method, which is based on simulated annealing method. This method is an algorithmic analog of physical process of controlled cooling and uses the ordered random search for new states of the system with a lower temperature [27].

During slow controlled cooling of the molten material, called annealing, the crystallization of the melt is accompanied by a reduction of its global power E, but the situation in which it can grow for a while (especially when heating the melt to prevent too rapid cooling) are implied. Thanks to the admissibility of a short-term increase in energy levels, there is a possible way out of the trap of local minimum energy that are arising in the implementation process. Only lowering the temperature T to absolute zero makes it impossible for any independent power increase of the melt.

Therefore, to develop a modified simulated annealing method must be defined with:

– the function of the energy E system,

– the function that describes the decrease of the temperature T over time,

– the function (rule) that creates a new state of the system.

In this case, the objective function of the task (2) will be the energy of the system, that is:

$$E = l(X), \qquad (9)$$

where: $X = \{x_1, y_1, \theta_1, \ldots, x_N, y_N, \theta_N\}$ – is the current state of the system.

As for the choice of function that describes the decrease of temperature T over time, historically the first simulated annealing scheme was Boltzmann's scheme in which the temperature change is:

$$T = \frac{T_0}{\ln(1+t)}, \qquad (10)$$

where: T_0 – is initial temperature; t – is time, $t > 0$.

For this scheme is proved that for sufficiently large values T_0 and the number of steps the finding a global minimum of the functions is guaranteed (9) [28] The disadvantage of Boltzmann's annealing is slow decreasing of the temperature T. The solving of this problem is possible by replacing the law of change of temperature (10), such as the following:

$$T_m = q\cdot T_{m-1},\ m = 1, 2, \ldots \qquad (11)$$

where: the temperature coefficient q is chosen as usual within $0{,}7 \div 0{,}99$. This scheme of simulated annealing allows to save the computing resources, but, while the finding a global minimum of the functions is not guaranteed (9).

As for the formation of a new state of system, then, the first, the random permutation of numbers of the placement objects is performed $\{i_1, i_2, \ldots, i_N\} \in \{1, \ldots, N\}$ and is determined by the current state of the system $X = \{x_1, y_1, \theta_1, \ldots, x_N, y_N, \theta_N\}$ through the consistent placement of objects according to permutation their numbers and taking into account the constraints (3)÷(5). Fig. 5 shows an example of placing an object $S_{i_2}\left(x_{i_2}, y_{i_2}, \theta_{i_2}\right)$ with fixed value θ_{i_2}.

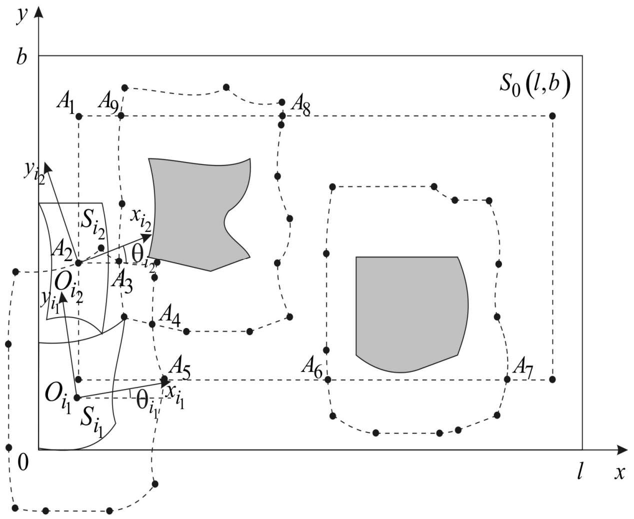

Fig. 5. Placement of object $S_{i_2}\left(x_{i_2}, y_{i_2}, \theta_{i_2}\right)$

Thus, in this case admissible points for placing the beginning of the local coordinate system of the object $S_{i_2}\left(x_{i_2}, y_{i_2}, \theta_{i_2}\right)$ is points $A_1, \ldots, A_9$, which are determined by means of second-order equations (both linear and nonlinear). The data of the system of equations are recorded by the corresponding transformation of inequalities (3)÷(5) to equalities. Obviously, in terms of impact on the value of the objective function (9) for the points A_1 and A_2 are equivalent to placement of the beginning of the local coordinate system of the object $S_{i_2}\left(x_{i_2}, y_{i_2}, \theta_{i_2}\right)$. However, based on the technological requirements we choose the point A_2. Likewise the placement of other objects is implemented $S_{i_j}\left(x_{i_j}, y_{i_j}, \theta_{i_j}\right)$, $j = 3, \ldots, N$, with the calculated objective function value $l(X)$ for the current state of the system.

For the formation of a new state of the system

$$X^* = \left\{x_1^*, y_1^*, \theta_1^*, \ldots, x_N^*, y_N^*, \theta_N^*\right\}$$

is possible, for example, to make the permutation of numbers of any two objects and carry out their placement according the new reshuffle considering the restrictions (3)÷(5). Further growth of energy system is computed:

$$\Delta E = l\left(X^*\right) - l(X).$$

If $\Delta E < 0$, then the system moves from state X to the state X^*. Otherwise, the transition to the state X^* is made with a probability $p\left(\frac{\Delta E}{T}\right)$, which is calculated:

$$p\left(\frac{\Delta E}{T}\right) = e^{-\frac{\Delta E}{T}}. \qquad (12)$$

Therefore, choosing the initial T_0 and final T_e temperature value T, that decreases over time, the optimization of the objective function of (9) is carried, with each successive generation of the system is subject to the limitations of (3) ÷ (5).

Obviously, to solve practical problems of optimal placement of the undirected planar geometrical objects with piecewise non-linear boundaries in the multiply area should be used the modified simulated annealing method, the complexity of which can be obtained using expressions (10) or (11) and is several orders of magnitude less than (8).

CONCLUSIONS

In this paper the problem statement is formulated and the mathematical model is created. The modified branch and bound method and modified method of simulated annealing to minimize the objective function for the problem of optimal placement undirected planar geometrical objects with piecewise non-linear boundaries in the multiply area are developed. It was shown that to solve practical problems it is advisable to use a modified simulated annealing method. Further research will be aimed at developing algorithmic and software to computer implementation of methods that are designed to optimize the placement of the planar undirected geometrical objects with piecewise non-linear boundaries of the multiply area.

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