UDK 614.8

Y.A. Abramov, DSc, Prof., chief researcher, NUCDU,
Y.Y. Kalchenko, Master, NUCDU

MATHEMATICAL MODELS FOR OBJECT TESTS OF THERMAL FIRE DETECTORS

The mathematical description of the processes in the sensor of thermal fire detectors under stationary heat impact of a standalone heat source is built.

Keywords: fire detector, heat flow, standalone heat source.

Problem formulation. Recently there has been a steady trend in increasing the role of mathematical models for object tests of fire detectors. One of the problems is the creation of mathematical models describing the thermal processes in the fire detectors when heat impact on the detector sensor is formed by standalone source.

Analysis of recent researches and publications. Mathematical models of object tests of fire detectors with thermoresistive sensor is the most developed [1, 2]. Integral transformation including fractional order Bessel function and integral Laplace transform are sequentially used for mathematical describing the heat processes in thermal fire detectors [3]. The double integral transformations complicate the algorithm for constructing the mathematical description of thermal processes.

Statement of the problem and its solution. The main goal of the work is to build mathematical models describing thermal processes in thermal fire detectors formed by standalone heat sources and using simplified algorithms.

Standalone method of forming the heat impact to fire detector sensor assumes that functional elements placed inside fire detector supply heat flow $q(t)$ to fire detector sensor.

Consider the case where $q(t) = q = \text{const}$.

The temperature distribution in a fire detector sensor will be described by Fourier equation

$$\frac{\partial T(x,t)}{\partial t} = a\frac{\partial^2 T(x,t)}{\partial x^2} - m^2\left[T(x,t) - T_0\right], \qquad (1)$$

with initial and boundary conditions

$$T(x,0) = T_0; \; \lambda\frac{\partial T(R,t)}{\partial x} = q, \qquad (2)$$

where a, λ are thermal diffusivity and thermal conductivity; T_0 – ambient temperature; m^2 is parameter defined by expression

$$m^2 = \frac{2\alpha}{c\rho R}; \tag{3}$$

c, ρ, R are specific heat capacity, density and thickness of the sensor; α is heat transfer coefficient.

Turn to the homogeneous initial conditions and introduce the notation

$$\theta(x,t) = T(x,t) - T_0, \tag{4}$$

and

$$\theta(x,t) = M(x,t)\exp(-m^2 t). \tag{5}$$

It transforms equation (1) to the form

$$\frac{\partial M(x,t)}{\partial t} = a\frac{\partial^2 M(x,t)}{\partial x^2}, \tag{6}$$

and conditions (2) to

$$M(x,0) = 0; \ \lambda\frac{\partial M(R,t)}{\partial x}\exp(-m^2 t) = q. \tag{7}$$

To solve the differential equation (6) with the conditions (7) apply to it the integral Laplace transform

$$\overline{M}(x,p) = \int_0^\infty M(x,t)\exp(-pt)dt.$$

It transforms (6) to the form

$$a\frac{d^2\overline{M}(x,p)}{dx^2} - p\overline{M}(x,p) = 0. \tag{8}$$

Taking into account the physical reasons allows to present the solution of the equation as

$$\overline{M}(x,p) = A(p)\exp\left(-x\sqrt{\frac{p}{a}}\right), \tag{9}$$

where $A(p)$ is integration constant.

Use boundary condition (7) to determine $A(p)$. Then

$$\frac{d\overline{M}(R,r)}{dx} = -\frac{q}{\lambda(p-m^2)} = -A(p)\sqrt{\frac{p}{a}}\exp\left(-R\sqrt{\frac{p}{a}}\right), \tag{10}$$

consequently

$$A(p) = \frac{q\sqrt{a}}{\lambda\sqrt{p}(p-m^2)}\exp\left(R\sqrt{\frac{p}{a}}\right). \tag{11}$$

In this case function $\overline{M}(x,p)$ can be written as

$$\overline{M}(x,p) = \frac{q\sqrt{a}}{\lambda\sqrt{p}(p-m^2)}\exp\left(-\frac{x-R}{\sqrt{a}}\sqrt{p}\right), \tag{12}$$

and its original takes the form

$$M(x,t) = L^{-1}[\overline{M}(x,p)], \tag{13}$$

where L^{-1} is operator of inverse integral Laplace transform operator.

To determine function $M(x,t)$ transform multiplier $[\sqrt{p}(p-m^2)]^{-1}$, which can be presented as follows

$$\frac{1}{\sqrt{p}(p-m^2)} = \frac{A}{\sqrt{p}} + \frac{B}{\sqrt{p}+m} + \frac{C}{\sqrt{p}-m}, \tag{14}$$

where A, B, C are coefficients determining by solution of the equation system

$$\begin{aligned} &A+B+C=0; \\ &B-C=0; \\ &Am^2=-1. \end{aligned} \tag{15}$$

Solving the system (15) allows to write (14) in the form

$$\frac{1}{\sqrt{p}(p-m^2)}=\frac{1}{2m^2}\left(\frac{1}{\sqrt{p}+m}+\frac{1}{\sqrt{p}-m}-\frac{2}{\sqrt{p}}\right). \quad (16)$$

Then (13) can be presented as follow

$$M(x,p)=\frac{q\sqrt{a}}{2m^2\lambda}L^{-1}[\overline{F_1}(x,p)+\overline{F_2}(x,p)-2\overline{F_3}(x,p)], \quad (17)$$

where

$$\overline{F_1}(x,p)=\frac{\exp(-K\sqrt{p})}{\sqrt{p}+m};\ \overline{F_2}(x,p)=\frac{\exp(-K\sqrt{p})}{\sqrt{p}-m}; \quad (18)$$

$$\overline{F_3}(x,p)=\frac{\exp(-K\sqrt{p})}{\sqrt{p}}.$$

There is $K=\frac{|x-R|}{\sqrt{a}}=\frac{R-x}{\sqrt{a}}$ in the expressions (18). It makes possible to write [4]

$$f_1(x,t)=L^{-1}\left[\overline{F_1}(x,p)\right]=\frac{\exp\left(-\frac{K^2}{4t}\right)}{\sqrt{\pi t}}-m\exp(Km+m^2t)\times \quad (19)$$

$$\times\,\mathrm{ertc}\left(\frac{K}{2\sqrt{t}}+m\sqrt{t}\right);$$

$$f_2(x,t)=L^{-1}\left[\overline{F_2}(x,p)\right]=\frac{\exp\left(-\frac{K^2}{4t}\right)}{\sqrt{\pi t}}+m\exp(-Km+m^2t)\times \quad (20)$$

$$\times\,\mathrm{ertc}\left(\frac{K}{2\sqrt{t}}-m\sqrt{t}\right);$$

$$f_3(x,t)=L^{-1}\left[\overline{F_3}(x,p)\right]=\frac{\exp\left(-\frac{K^2}{4t}\right)}{\sqrt{\pi t}}. \quad (21)$$

Taking the (5), (17) and (19)$\div$(21) into account means that

$$\theta(x,t)=\frac{q\sqrt{a}\exp\left(-\frac{R-x}{\sqrt{a}}m\right)}{2m\lambda}[\mathrm{ertc}\left(\frac{R-x}{2\sqrt{at}}-m\sqrt{t}\right)- \quad (22)$$

$$-\exp\left[\frac{2(R-x)m}{\sqrt{a}}\right]\mathrm{ertc}\left(\frac{R-x}{2\sqrt{at}}+m\sqrt{t}\right)].$$

For the steady state, ie $\theta_{уст}(x)$ it can be written

$$\theta_{st}(x) = \lim_{t\to\infty}\theta(x,t) = \frac{q\sqrt{a}}{m\lambda}\exp\left(-\frac{R-x}{\sqrt{a}}m\right). \qquad (23)$$

Because output signal of thermoresistive fire detector sensor is temperature averaged over its volume, it takes place

$$\theta_{st} = \frac{1}{R}\int_0^R \theta_{st}(x)dx = \frac{q\sqrt{a}\exp\left(-\frac{mR}{\sqrt{a}}\right)}{m\lambda R}\int_0^R \exp\left(\frac{mx}{\sqrt{a}}\right)dx =$$
$$= \frac{qa}{m^2\lambda R}\left[1-\exp\left(-\frac{mR}{\sqrt{a}}\right)\right]. \qquad (24)$$

Expression (3) and

$$a = \frac{\lambda}{c\rho}, \qquad (25)$$

make it possible to write (24) as follows

$$\theta_{st} = \frac{q}{2\alpha}\left[1-\exp\left[-\left(\frac{2\alpha R}{\lambda}\right)^{0,5}\right]\right]. \qquad (26)$$

Inequality

$$\frac{2\alpha R}{\lambda} << 1, \qquad (27)$$

means that expansion of the exponent function in a power series leads to

$$\theta_{st} \cong q\left(\frac{R}{2\alpha\lambda}\right)^{0,5}. \qquad (28)$$

Consequently, when heat flow impact $q = const$ at fire detector sensor is in a steady state, sensor temperature averaged over its volume is more than initial temperature by value which is proportional to the magnitude of the heat flow. This fact can be used to form an algorithm of site-testing the thermal fire detectors. Presence of standalone heat sources has to be provided in this case.

Conclusions. The problem of describing the processes occurring in fire detectors when they are impacted by stationary heat flow from standalone heat source is solved by using the integral Laplace transform. It is shown in this case the temperature increment of the fire detector is proportional to the magnitude of the heat flow.

LITERATURE

1. Abramov Y.A. Temperature site-testing the thermal fire detectors with thermoresistive sensor / Y.A. Abramov, V.V. Kovregin, V.P. Sadkovoy. – Kharkiv: UCDU, 2009. – 115 p.

2. Abramov.Y.A. Methods of determining the time constant of the thermal fire detectors with thermoresistive sensor / Y.A. Abramov, A.E. Basmanov, V.M. Gvozd. – Kharkiv: NUCDU, 2013. – 151 p.

3. Abramov Y.A. Software for autonomous method of site-testing the heat fire detectors. / Y.A. Abramov, Y.Y. Kalchenko // Problems of fire safety. – Kharkiv.: NUCDU, 2015. – V. 38. – P. 3-8.

4. Bateman H. Tables of integral transforms. Fourier transform, Laplace transform, Mellin transform / H. Bateman, A. Erdelyi. – Moscow: Science, 1969. – 344 p.

Ю.А. Абрамов, Я.Ю. Кальченко

Математическое обеспечение алгоритма проведения объектовых испытаний тепловых пожарных извещателей

Получено математическое описание для процессов, протекающих в чувствительном элементе тепловых пожарных извещателей при стационарном тепловом воздействии, создаваемом автономным источником тепла.

Ключевые слова: пожарный извещатель, тепловой поток, автономный источник тепла.

Ю.О. Абрамов, Я.Ю. Кальченко

Математичне забезпечення алгоритму проведення об'єктових випробувань теплових пожежних сповіщувачів

Отримано математичний опис для процесів, що протікають в чутливому елементі теплових пожежних сповіщувачів при стаціонарному тепловому впливі, що створюється автономним джерелом тепла.

Ключові слова: пожежний сповіщувач, тепловий потік, автономне джерело тепла.